Mechanism Design

4: Correlated Information

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Independent vs Correlated Information

- So far we always assumed types θ_i independently distributed.
  - Not an innocuous assumption.
  - Very important for results regarding BIC mechanisms (DSIC are not affected).
- What if types are correlated? Does it hurt or help the designer?
  - If θ_i are correlated then each i has some information about all θ_j.
  - Can use it to cross-check reports of θ_j.

This slide deck:

1. Perfect correlation
2. Imperfect correlation

Perfect Correlation

- Consider a modified quasilinear setting.
- N agents with perfectly correlated types: $\theta_i = \omega \ \forall i$.
- Designer does not know $\omega \in \Omega$, only knows the distribution $F(\omega)$ (as usual).
- Designer wants to implement some allocation $k(\omega)$.
- **Example:** a common-value auction, where all buyers have the same valuation for the item, but the seller is unaware of this valuation. (E.g., your parents selling your collection of rare pokemon cards on ebay.)

Perfect Correlation

- Consider the following direct mechanism:
  - If all agents' reports agree ($\hat{\theta}_i = \hat{\theta}_j = \hat{\omega}$ for all i, j) then implement $k(\hat{\omega})$ with zero transfers.
  - Otherwise implement any $k \in K$ and set $t_i = +\infty$ for all i.
  - (If the desired s.c.f. $k(\theta)$ prescribes an allocation to mismatching types, can use that instead.)
- This mechanism truthfully implements $k(\omega)$ (BIC? DSIC?).
  - Never profitable to deviate alone (then must pay infinity).
  - There are also many other equilibria apart from truthful one...

Q: In truthful eqm, would $t_i = +\infty$ after disagreement interfere with IR? (if we care about IR)

This slide deck:

1 Perfect correlation

2 Imperfect correlation

Imperfect correlation: setup

- Still a modified quasilinear world.
- N agents with private types $\theta_i \in \Theta_i$ (Θ_i finite).
- Types have some joint distribution: $(\theta_1, ..., \theta_N) \sim \phi(\Theta)$ (p.d.f.).
- Player i's beliefs about θ_{-i} are derived by Bayes' rule:

$$\phi(\theta_{-i}|\theta_i) = \frac{\phi(\theta_{-i}, \theta_i)}{\sum_{\theta'_{-i} \in \Theta_{-i}} \phi(\theta'_{-i}, \theta_i)}$$

- Designer only knows the distribution ϕ, wants to implement some allocation $k(\theta)$.

Imperfect correlation: a simple example

Example (2x2)

	H	L
H	$\frac{1}{6}$	$\frac{1}{3}$
L	$\frac{1}{3}$	$\frac{1}{6}$

- 2 agents, 2 types each: $\theta_i \in \{H, L\}$, joint distribution $\phi(\theta_1, \theta_2)$ in the table above.
- Each agent thinks $\theta_j = \theta_i$ is twice less likely than otherwise.

Imperfect correlation

- A similar idea can be used as with perfect correlation.
- Now i does not know θ_j perfectly – can't use this info to cross-verify.
- But can force i to gamble on θ_j.

Cremer-McLean condition

- Write your beliefs $\phi(\theta_{-i}|\theta_i)$ as a vector $\tilde{\phi}(\theta_i)$ (for each $\theta_{-i} \in \Theta_{-i}$ one entry in the vector).
- Cremer-McLean condition: no such vector $\tilde{\phi}(\theta_i)$ is a linear combination of the other vectors in $\{\tilde{\phi}(\theta_i') : \theta_i \neq \theta_i' \in \Theta_i\}$

Definition (CM condition)

The distribution ϕ satisfies the CM condition if there are no agent i with type $\theta_i \in \Theta_i$ and weights $\lambda_i : \Theta_i \setminus \{\theta_i\} \to \mathbb{R}_+$ such that

$$\phi(\theta_{-i}|\theta_i) = \sum_{\theta_i' \in \Theta_i \setminus \{\theta_i\}} \lambda_i(\theta_i')\phi(\theta_{-i}|\theta_i') \qquad \text{for all } \theta_{-i} \in \Theta_{-i}.$$

Cremer-McLean condition

- Stack vectors $\tilde{\phi}(\theta_i)$ for each type θ_i into a matrix.
- The condition holds if this matrix has full rank.
- In particular, every type θ_i must have its own distinct belief about the distribution of θ_{-i} (but the condition is stronger than this).
- Does the condition hold in the 2x2 example?
- How about in the following:

Example (2x3)

	H	M	L
H	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{6}$
L	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$

Cremer-McLean result

Theorem (Cremer-McLean)

If ϕ satisfies the CM condition, then for any mechanism (k, t) there is a direct mechanism (k, t') such that

- *(k, t') is BIC,*
- *both mechanisms have the same allocation k,*
- *both mechanisms have the same interim expected payoffs: $\forall i, \theta_i$,*

$$\mathbb{E}_{\theta_{-i}}\left[t_i(\theta)|\theta_i\right] = \mathbb{E}_{\theta_{-i}}\left[t_i'(\theta)|\theta_i\right]$$

- Holds for any mechanism (k, t) – IC not required!
- Meaning any allocation rule k is BIC if ϕ satisfies CM condition!

Proof Idea

Forget about the allocation k and think about how to elicit private information truthfully.

Example (Information elicitation)

- State of the world $\omega \in \Omega$; expert knows true distribution π of states (but not the state); designer knows nothing.
- How to extract information about π from the expert?
- Consider the following scheme:
  - the expert announces a probability distribution $\hat{\pi}$;
  - when state ω realizes, the expert is paid $\log(\hat{\pi}(\omega))$

$$\text{Expert's problem: } \max_{\hat{\pi}} \sum_{\omega \in \Omega} \pi(\omega) log(\hat{\pi}(\omega))$$

$$\text{subject to } \sum_{\omega \in \Omega} \hat{\pi}(\omega) = 1.$$

- Solve using Lagrange method to see that truthful reporting is optimal.

Proof Idea

Proof idea

- In our mechanism design problem, consider transfers

$$t_i'(\theta_i, \theta_{-i}) = t_i(\theta_i, \theta_{-i}) + C_{i,1} - C_{i,2} \log(\phi(\theta_{-i}|\theta_i)).$$

- If $C_{i,2}$ is large enough, i's incentives are dominated by the need to report $\phi(\theta_{-i}|\theta_i)$ correctly (rather than desire to get best $k(\theta)$).
- So set $C_{i,2}$ large, then use $C_{i,1}$ to adjust the averages as required and voila. "$\Box$"

- The above is not a complete proof, since we would actually want $C_{i,1}$ to depend on θ_i to get $\mathbb{E}_{\theta_{-i}}[t_i(\theta)|\theta_i] = \mathbb{E}_{\theta_{-i}}[t_i'(\theta)|\theta_i]\ \forall i, \theta_i$. Otherwise we only have $\mathbb{E}_\theta[t_i(\theta)] = \mathbb{E}_\theta[t_i'(\theta)]$ $\forall i$.
  - But making $C_{i,1}(\theta_i)$ depend on θ_i affects reporting incentives...
- For full proof (and construction) see Börgers ch6.4 or Cremer & McLean (1988).

Conclusion

- Cremer-McLean result is a VERY powerful tool to implement literally anything under correlated information.
- **Issues:**
- Strong-ish condition on ϕ –
  - this method cannot extract private information which does not affect i's belief about θ_{-i}.
  - But we can use it as a first step to extract some info, then proceed as before.
- With weak correlation $C_{i,2}$ can be HUGE,
  - leading to extremely large (positive and negative) $t_i(\theta)$.
  - Not good if want *ex post* IR, *ex post* BB, and/or *limited liability* ($t_i(\theta) \leq 0$ – reasonable requirement in some settings, close to ex post IR).

Correlated information: Conclusion

- Correlated information can be **very** easily exploited
  - Make the players snitch about each other's types!
  - We'll see an example later where if players' preferences conflict, we can set them against each other (divide et impera!).
- Of course there are always issues that can invalidate our analysis:
  - Risk aversion, collusion, competition, limited liability, budget constraints...
  - Designer may not know the exact beliefs that every type has. See Lopomo, Rigotti, and Shannon (2020) for analysis of this case and references to papers dealing with the above cases.
  - Börgers ch.10 takes a more general approach to designer's uncertainty about players' beliefs ("Robust Mechanism Design")