# Exercises for Lecture 9 (M6): Dynamic mechanisms without transfers.

## Problem 1: Parenting

This problem asks you to solve a simplified version of the model presented in Li, Powell, and Matouschek (2017). For a change, we will consider a story that is different from theirs. Suppose that parents (the designer) are debating with their teenage kid (the player/agent) regarding the kid's possible career paths. In particular, in every period $t \in \{0, 1, \dots, \infty\}$ the kid can take up one of three jobs, $a \in \{l, m, r\}$, which are as follows:

- $a = l$ means beginning/continuing on track to becoming a lawyer, whether it is studying or taking up appropriate jobs. This yields payoff $B$ to the parents and $b$ to the kid.
- $a = m$ means working at a local McDonald's™, which yields payoff 0 to both parents and the kid.
- $a = r$ means playing rock music. This yields payoff $B$ to the kid, who really enjoys playing rock music, but only yields payoff $b$ to the parents, who would prefer the kid to become a lawyer, but agree that rock music is better than working at McDonald's™.

The assumption in the above is that $B > b > 0$.

Being a good lawyer requires inspiration, which may or may not be present in any given period. Let the state $\theta_t \in \{0, 1\}$ denote whether the kid has lawyer's inspiration in period $t$. If $\theta_t = 0$ then $a = l$ is not a feasible choice – i.e., it can not be chosen in period $t$.[1] The common belief of the parents and the kid is that $\theta_t$ is i.i.d. across periods, and $\mathbb{P}\{\theta_t = 1\} = \phi$. Once period $t$ arrives, the kid privately observes $\theta_t$ before making a decision, but the parents do not observe $\theta_t$, at $t$ or afterwards.

The parents can put a hard veto on whether their kid becomes a rock musician in any given period, $s_t \in \{0, 1\}$. I.e., they can exclude $a = r$ from the choice set. If this happens ($s_t = 1$), the kid then faces a choice between $a_t \in \{l, m\}$ if $\theta_t = 1$ and $a_t = m$ (no choice) if $\theta_t = 0$, while if the parents give the kid the freedom to choose ($s_t = 0$), then the choice is between $a_t \in \{l, m, r\}$ if $\theta_t = 1$ and $a_t \in \{m, r\}$ if $\theta_t = 0$.

Assume that both the parents and the kid are forward-looking and discount the future using discount factor $\delta \in (0, 1)$.[2] Your goal is to design an optimal veto strategy for the parents.

1. Consider the parents' strategy of never imposing the veto power ($s_t = 0$ for all $t$). What is the kid's optimal strategy then? (Note that a strategy must specify an action for every history – i.e., for every period $t$, every realization of $\theta_t$, given every possible history of past states and actions.) Calculate the parents' expected discounted lifetime payoff $V_p^{free}$ from the kid following this optimal strategy. Calculate the kid's expected discounted lifetime payoff $V_k^{free}$.

   *Note: we think of these payoffs as being estimated before the first state $\theta_0$ is revealed to the kid.*

2. Consider the parents' strategy of always restricting the kid ($s_t = 1$ for all $t$). Answer the same questions as in part 1: what is the kid's optimal strategy? What is the parents' payoff $V_p^{veto}$? What is the kid's payoff $V_k^{veto}$?

Consider now the following strategy for the parents. In the first period, $t = 0$, they give the kid the freedom of choice ($s_0 = 0$). If $a_0 = l$, the parents never control the kid again $s_t = 0$ for all $t \geq 1$). If $a_0 \in \{r, m\}$, the parents will always control the kid ($s_t = 1$ for all $t \geq 1$). The intent is that this will incentivize the kid to choose $a_0 = l$ if $\theta_0 = 1$. Assume also that parents can commit to such "conditional veto" strategy at $t = 0$.

[1] You can think that choosing $a_t = 0$ when $\theta_t = 0$ yields utility $-\infty$ to the kid.
[2] One util tomorrow is worth $\delta$ utils today.

3. Derive the IC condition for the kid at $t = 0$ (for it to be optimal for them to choose $a_0 = l$ if $\theta_0 = 1$ under the conditional veto strategy).
4. Calculate the parents' expected utility from adopting the conditional veto strategy (assuming the kid's IC condition holds).
5. Assume the following parameter values: $\phi = 1/2$, $\delta = 1/3$, $B = 3$, $b = 2$. Verify that the kid's IC condition holds.
6. Given the parameter values from part 5: is this strategy better for the parents than laissez-faire (no veto ever) and/or permanent restriction, from parts 1 and 2 of this problem respectively?
7. Given the parameter values from part 5: argue to the best of your ability what the parents' optimal strategy is. (If they can do better than conditional veto, explain how. If they can not, explain why.)