Extracting Correlated Information Without Transfers

This note explains how equilibrium works in the model of Battaglini (2003) and how to find it. For model setup, see M4 slide deck.

The equilibrium is described by a profile of strategies for all players, (a, m_1, m_2), where $a(m_1, m_2)$ is the principal's choice of action given messages, and $m_i(\omega)$ is the agent i's choice of message given the true state. The principal chooses a to maximize his expected payoff $\mathbb{E}_\omega[u_p(a,\omega)|m_1,m_2]$ conditional on the two reports (m_1, m_2), where $u_p(a,\omega) = -\left(\|a-\omega\|_2\right)^2$. Therefore, his equilibrium strategy must be given by

$$a^*(m_1, m_2) \equiv \mathbb{E}\left[\omega|m_1, m_2\right] \tag{1}$$

In other words, by changing their reports m_i, the agents can directly affect the principal's choice of action. Our mission of finding a fully revealing equilibrium thus lies in designing communication strategies $m_i(\omega)$ in such a way that:

1. $a^*(m_1(\omega), m_2(\omega)) = \omega$, i.e. the two messages allow to identify the state;
2. $m_i(\omega) \in \arg\max_{\hat{m}_i} \left\{-\left(\|a^*(\hat{m}_i, m_j(\omega)) - (\omega + b_i)\|_2\right)^2\right\}$, i.e. $m_i(\omega)$ should be optimal for agent i.

Look at Figure 1. What is the set of actions available to player 1 conditional on player 2 reporting truthfully? In other words, what actions a can player 1 induce through various reports $\hat{m}_1$, given that player 2 follows the prescribed strategy $m_2(\omega)$ (we do not know how it looks yet). For agent 1 to be willing to report truthfully, leading to action $a = \omega$, he must weakly prefer that over all other actions a he can induce – meaning that the set of such actions must lie outside of the outer blue circle in Figure 1. In particular, it would be fine if the set of such actions was given by the purple line l_1, which is orthogonal to b_1. Similarly,

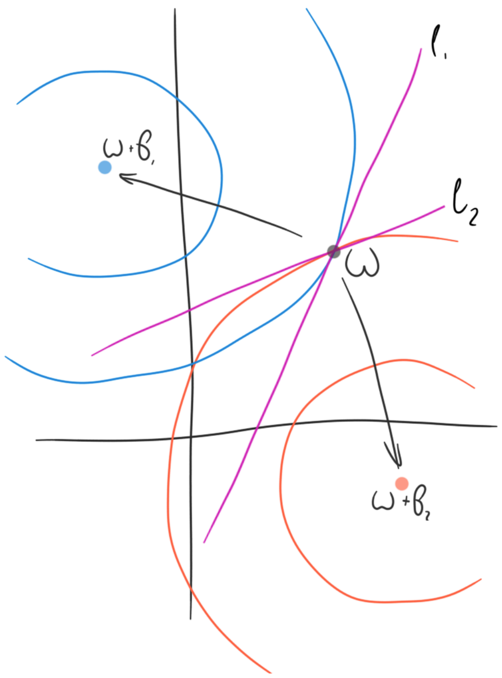

Figure 1: Figure 1

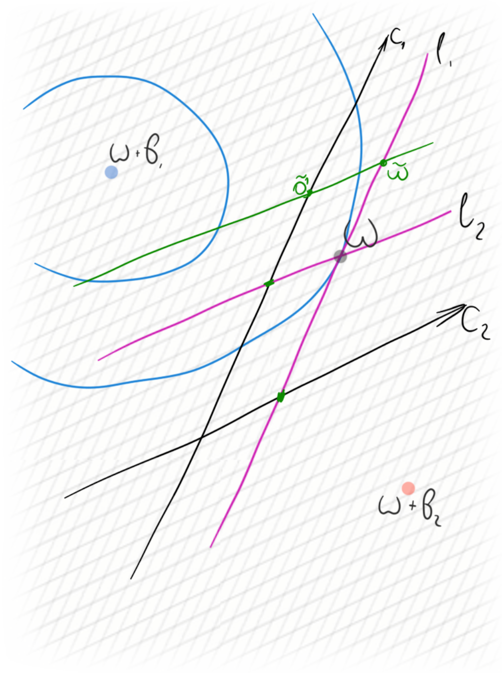

Figure 2: Figure 2

the set of actions available to player 2 in equilibrium conditional on P1 following $m_1(\omega)$ can be given by l_2, among others.

The question is how we determine the *locations* of these respective lines/action sets. One might think of a kind of two-stage communication mechanism: in stage 1, P1 reports where l_2 should be and vice versa, and in stage 2, both players i report where the state is on their respective lines l_i. Note, however, that the second stage is redundant: once we know where l_1 and l_2 are, we already know the state ω (hint: it is the unique point at which the two lines intersect). But will it actually be an equilibrium for player i to correctly report the location of l_j? In fact, yes, since by misreporting player i will only be able to induce actions on l_i, which contains no better options for i than the truth. Let us now construct the equilibrium formally.

Consider a basis (c_1, c_2), where c_i is a vector orthogonal to b_i for each i.[1] So long as b_1 and b_2 are linearly independent, this basis generates the whole linear plane. This means, in particular, that any state $\omega = (\omega^1, \omega^2) \in \mathbb{R}^2$ can be uniquely represented in this basis in terms of pair of coordinates (o^1, o^2), i.e.,

$$\omega = o^1 \cdot c_1 + o^2 \cdot c_2$$

(where ω, b_1, b_2 are all vectors).

Now consider a mechanism in which each player i reports o^i.[2] The principal then takes action $a(o^1, o^2) = o^1 \cdot c_1 + o^2 \cdot c_2$. This decision rule trivially satisfies the principal's optimality condition (1) if agents report o^1 and o^2 truthfully, so we are only left to verify that truth-telling is indeed optimal.

This is best illustrated graphically. Look at Figure 2. Suppose that P1 reported some $\tilde{o}^1 > o^1$. This would shift the induced action upwards along l_1 to $\tilde{\omega}$. We have chosen l_1 in such a way that any action on it is worse for P1 than that induced by the truth, hence this deviation is not beneficial for P1. Similar logic applies to any other deviation at the communication stage.

[1] E.g., letting $c_i = \frac{1}{\|b_i\|_2} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot b_i$ would yield a vector c_i of unit length that is rotated 90 deg clockwise w.r.t. b_i.

[2] Formally, the problem setup says that messages are two-dimensional: $m_i \in \mathbb{R}^2$. So to be 100% formal you can say that, e.g., $m_i = (o^i, 0)$, and that the principal ignores the second coordinate of each message.