Mechanism Design

0: Introduction

Egor Starkov

København Unversitet
Fall 2024

This slide deck:

1. What is mechanism design?
2. Logistics
3. First Taste

History of (Micro)Economic Thought (abridged)

Question	Answer	Tool

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How well do markets work?		

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How to fix the inefficiencies?	We'll see!	Mechanism design!

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You want to sell an apartment. What is the best mechanism to do so?

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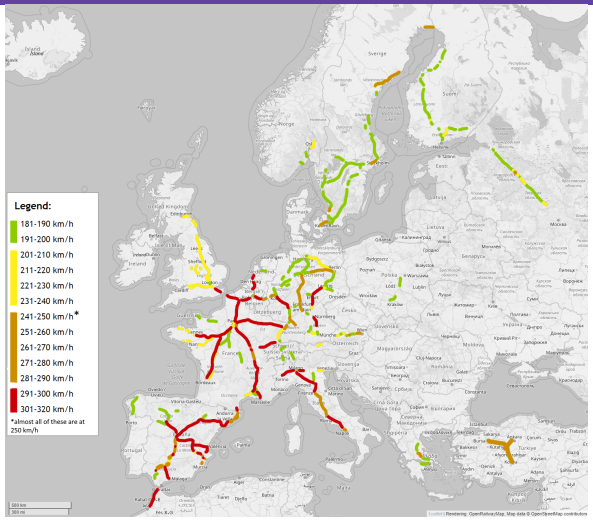
- Even setting a take-it-or-leave-it price defines some decision problem for a buyer (which price to set then?)
- Can bargaining perform better than a fixed price?
- Should you try to attract more buyers? (How?)
- How much info about the apartment do you reveal?

Auctions, optimal pricing, and related questions are a large part of mechanism design.

Example 2

Suppose EU wants to connect its high-speed rail networks. How should the costs be split across countries?

- Proportional to distance of lines built?
- Proportional to costs of lines built?
- Proportional to benefits accrued?
- ...how to elicit whichever is chosen?



Example 3

How to allocate students to schools?

- Parents/students have some preferences over schools,
- Schools might have priorities over students,
- Impossible to respect all preferences.
- And we can't use money.
- How to organize the application/allocation process to get the best outcome?

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- Eliciting hidden information
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> Last year, another Microsoft survey found that the typical worker using its software spent 57 percent of their time "communicating"—that is, in meetings, email, and chat—versus 43 percent of their time "creating" documents, spreadsheets, presentations, and the like. Today, knowledge work is, quantitatively speaking, less about creating new things than it is about *talking about* those things.

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  - Will also look at the complementary question: what's the optimal way to supply information?
- Hidden action problems (inducing, monitoring, enforcement)
  - We won't look at those, see **contract theory** course
- Aggregating social preferences / sharing the surplus
  - We'll mostly follow the utilitarian approach. See *social choice theory* for more approaches.
  - Cooperative game theory offers alternative approach to surplus sharing problems.
  - See also the **distributive justice** course.

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  - What's the solution to the prisoner's dilemma? To the lemons market game?
  - What's the solution to social polarization? To unemployment? To war?

What is this course? Methods

- Most MD problems can be reduced to some maximization problem, but a simple Largangian is rarely enough...
- (due to high dimensionality of the problems and/or high number of specific constraints)
- So we'll also be looking at some tools and methods that help solve mechanism design problems.
- Can't hope to cover the whole universe of problems, so we'll only look at selected few.

What is this course?

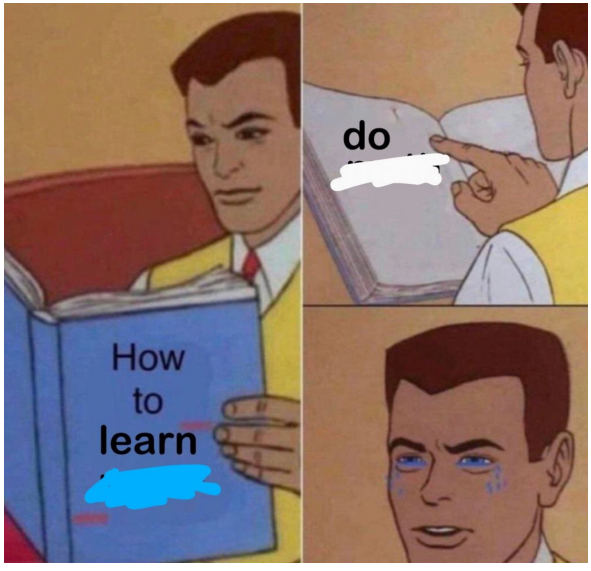
What can you expect?

- a crash course on formalizing problems
- overview of classic results (problems and solutions) over past 40 years
- a bit from the frontier but not much
- and...

...and we need to talk about math

Math

- There's a lot of math in this class
- Remember: **you don't suck at math!**
  - No one is inherently good or bad at math!
- Math is just a language you've got to learn
- It's difficult, so it's easy to get discouraged
  - And the defeatist moods are self-reinforcing
- And the only way into it is practice
  - Believe in yourself and never give up!


How
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learn
do

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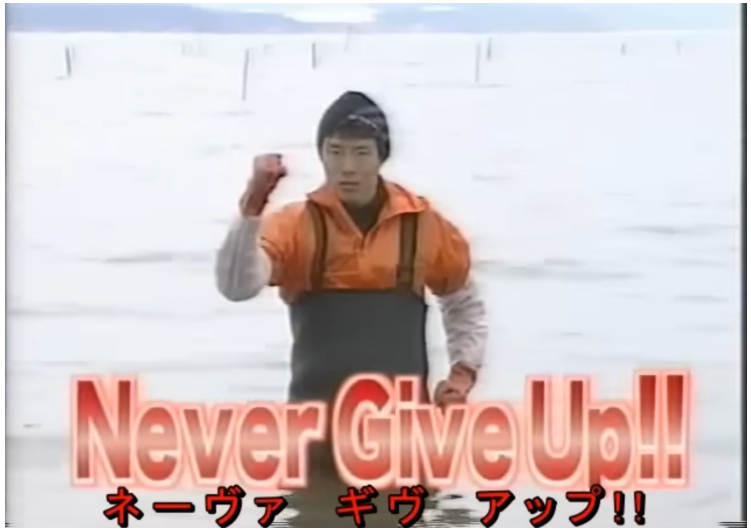

(click this pic every time you feel like you can't do it any more)

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Hi

- Egor Starkov
- Contact: egor.starkov@econ.ku.dk or absalon inbox
- Research interests: information economics, dynamic games, communication
- Office: 26.1.13
- Office hours: Tue, 14-15
- Questions: email/absalon, before/after class

What about you?

Logistics

- Weekly lectures (except Fall break – week #42)
  - Tue, 15:15-18:00, CSS 35.0.12 (but check timetable for room changes)
  - talk later about what happens in those

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  - Formalize problems; solve them and explain intuition
- Weekly problem sets (ungraded, for practice)
- Research module for PhD students: contact me if you would like to do it

Workflow

- Intended workflow:
  1. We start a topic in class (very briefly)
  2. You watch video-lectures at home during the week (and/or read textbook, slides, papers)
  3. We go through the same material **very quickly** in class and discuss any questions you have
  4. We solve some problems in class
  5. You have more problems to practice at home

Have you noticed the emphasis above? If you don't prepare for lectures, life will be difficult!

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- Suggestion: organize into study groups. Watch videos in groups. Discuss problems in groups. Let me know by Friday if you want to join a group (assignment on absalon).
- In class I use whiteboard+slides for "lecture" stuff. Slides on absalon include some of the whiteboard parts.
  - I'll upload slides in advance, but they might be edited and updated afterwards

The problems are whiteboard-only; with solutions (mostly) uploaded afterwards.

References: textbooks

This course is a compilation of many books, papers, courses; does not follow any single one too closely. Below are some books that might help (see the reading list on Absalon for full references). Note that notation will be different across books and the class!

- **Narahari**: Probably your best bet. Hard to find in print, but you have online access through the library (see Absalon).
- **Diamantaras**: Another good textbook, but seems very hard to find.
- **Börgers**: I used this as default in previous years, but it's quite hardcore and hard to follow. (Easy to find though.)
- **MWG**: A microeconomic bible. Very good, very clear, but has the smallest coverage for our course.
- **RS**: Relevant for two lectures on matching. Some material is contained in Narahari and Diamantaras. Nice reference if you are into matching.

References: other

- I will sometimes refer to individual papers and surveys for results outside of textbooks.
- Some of these are completely optional
- Some I expect you to know (but try to explain well enough in the slides).
- See the reading list on Absalon for details (will likely be updated during the course).

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First taste: Communication problem

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- let $\theta \in \Theta$ denote the state of the world, known to the agent
- let $a \in A$ denote the principal's final action
- suppose the principal asks the agent to send message m about the state
- principal cannot verify whether the message/report is truthful or not

First taste: Communication problem

Communication problem

Can the principal *elicit* the agent's private information?

- In other words, will the agent report truthfully?
- The answer depends on the nature of the relationship
  - What will the principal do with this knowledge?
  - How does the agent feel about that?
  - Let's look at a few examples...

Common interests

Example 1: Common interests

A tourist (principal) is asking a person on the street (agent) which way to the mermaid statue

(u_P, u_A)	$\theta = l$	$\theta = r$
$a = l$	$2, 1$	$0, 0$
$a = r$	$0, 0$	$2, 1$

- where $\theta \in \{r, l\}$ is the state of the world (the correct direction)
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- Suppose the principal believes and follows the report: $a(m) = m$
- The agent would then truthfully report the state: $m(\theta) = \theta$, no reason to lie.

Opposed interests

Example 2: Opposed interests

A judge (principal) is asking a suspect (agent) whether they are guilty of a crime

(u_P, u_A)	$\theta = n$	$\theta = g$
$a = n$	$1, 2$	$0, 2$
$a = c$	$0, 0$	$1, 0$

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- If the principal believes the report ($a(m) = m$), the agent would lie and report $m(\theta) = n$ regardless of θ.
- Otherwise, whatever the principal's strategy $a(m)$ is, if there is some report m that leads to $a(m) = n$, the agent would always use it, regardless of θ!
- There is no way for the principal to get different reports in $\theta = n$ and $\theta = g$!

Partially aligned interests

Example 3: Partially aligned interests

A president (principal) is asking an economic advisor (agent) whether to enact a tax reform

(u_P, u_A)	$\theta = l$	$\theta = n$	$\theta = h$
$a = n$	$4, 1$	$2, 2$	$0, 1$
$a = r$	$2, 0$	$4, 1$	$2, 2$

- $\theta \in \{l, n, h\}$ is the state of the economy (need for a reform), equal probabilities
- $a \in \{n, r\}$ is the principal's action (whether to reform), wants $a = r$ if $\theta \in \{n, h\}$
- the agent knows θ, wants $a = r$ only if $\theta = h$

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- if the principal believes $m = \theta$, then best response is $a(l) = n$, $a(n) = a(h) = r$
- the agent is then partially honest: $m(l) = m(n) = l$, $m(h) = h$!
- $a(l) = n$ is still a best response for the principal, even if they understand the deception

Takeaways

- Agent is willing to reveal their private info if and only if it is in their interest to do so.
- Can we do better than "just asking"?

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- Agent is willing to reveal their private info if and only if it is in their interest to do so.
- Can we do better than "just asking"? Turns out, NO.
- ...But if action is multi-dimensional, can often find partial alignment and elicit at least some information.
  - E.g., another dimension could be a payment

Partial alignment in multidimensional settings

Example 3b: Partially aligned interests + payments

(v_P, v_A)	$\theta = l$	$\theta = n$	$\theta = h$
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- Suppose now P chooses $a(m)$ together with payment $t(m)$ from A; payments are linearly added to/subtracted from utilities: $u_P = v_P + t$, $u_A = v_A - t$

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- Suppose P "fines" the agent for advising against the reform: $t(l) = 1$, $t(n) = t(h) = 0$
  - (or, equivalently, think of rewarding the agent for advising the reform: $t(l) = 0$, $t(n) = t(h) = -1$)

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  - (or, equivalently, think of rewarding the agent for advising the reform: $t(l) = 0$, $t(n) = t(h) = -1$)
- Then (check!) there exists an equilibrium with truthful communication!

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- Agent is willing to reveal their private info if and only if it is in their interest to do so.
- Can we do better than "just asking"? Turns out, NO.
- ...But if action is multi-dimensional, can often find partial alignment and elicit at least some information.

For next week

1. Split into study groups (if you are looking for a group: respond to an assignment on absalon by Friday or email me)
2. Have a look at math review notes on absalon
3. Watch lectures 2.1 ('What is a mechanism?') and 2.2 ('Dominant strategy implementation') on youtube.
   - Or read Narahari ch.14-16