

Mechanism Design

0: Introduction

Egor Starkov

Københavns Universitet
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This slide deck:

1 What is mechanism design?

2 Logistics

3 First Taste

What is Game Theory?

Economic agents interact with each other.

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Economic agents interact with each other.



What is the outcome?

How is it shaped by environment?

What is Mechanism Design?

There is some desirable outcome.

What is Mechanism Design?

How to shape the environment to achieve it?
(in a strategic setting or a decision problem)

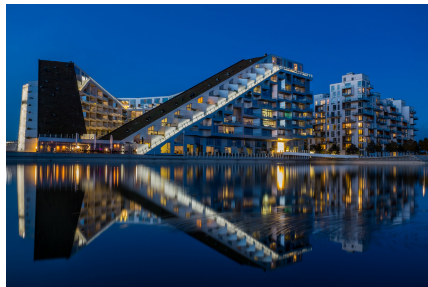


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Example 1

You want to sell an apartment. What is the best **mechanism** to do so?

- Even setting a **take-it-or-leave-it price** defines some decision problem for a buyer (which price to set then?)



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You want to sell an apartment. What is the best **mechanism** to do so?

- Even setting a **take-it-or-leave-it price** defines some decision problem for a buyer (which price to set then?)
- Can **bargaining** perform better than a fixed price?
- Should you try to attract more buyers? (How?)
- How much info about the apartment do you reveal?

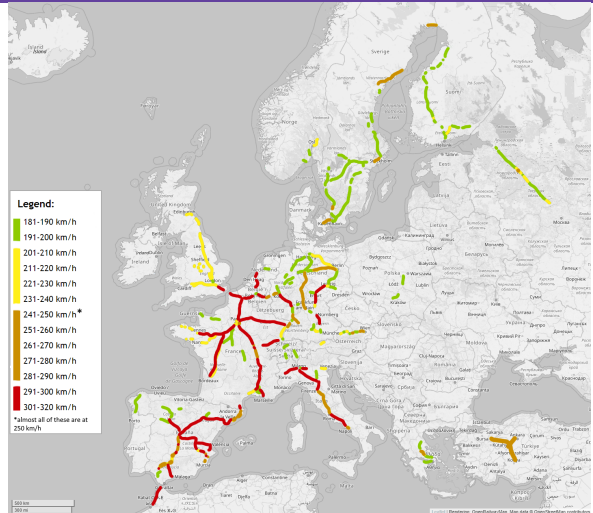
Optimal pricing and related questions is a large share of mechanism design.



Example 2

Suppose EU wants to connect its high-speed rail networks. How should the costs be split across countries?

- Proportional to distance of lines built?
- Proportional to costs of lines built?
- Proportional to benefits accrued?
- ...how to elicit whichever is chosen?



Example 3

How to allocate students to schools?

- Parents/students have some preferences over schools,
- Schools might have priorities over students,
- Impossible to respect all preferences.
- How to organize the application/allocation process to get the best outcome?



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 - formulating an objective;
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 - What's the solution to $x^2 = 4$? To $ax^2 + bx + c = 0$?

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 - What's the solution to the prisoner's dilemma? To the lemons market game?
 - What's the solution to social polarization? To unemployment? To war?

What is mechanism design? Problems

We'll be talking a lot about [asymmetric information](#). Why?

- First Welfare Theorem: competitive markets are efficient and cool and the best.
- So for interesting settings, have to look beyond Walrasian markets
- (asymmetric information, monopoly power, externalities, etc...)

What is mechanism design? Problems

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- First Welfare Theorem: competitive markets are efficient and cool and the best.
- So for interesting settings, have to look beyond Walrasian markets
- (asymmetric information, monopoly power, externalities, etc...)
- Many [economic problems](#) deal with asymmetric information:
 - eliciting a potential buyer's willingness to pay,
 - eliciting students' preferences over schools,
 - eliciting voters' preferences over candidates,
 - etc...

What is mechanism design? Methods

- Most MD problems can be reduced to some **maximization problem**, but a simple Lagrangian is rarely enough...
- (due to high dimensionality of the problems and/or high number of specific constraints)
- So we'll also be looking at some **tools and methods** that help solve mechanism design problems.
- Can't hope to cover the whole universe of problems, so we'll only look at selected few.

What is mechanism design? This course

What can you expect?

- a crash course on formalizing problems
- overview of classic results (problems and solutions) over past 40 years
- a bit from the frontier but not much
- and...

...and we need to talk about math



- There's a lot of math in this class
- I know most of you suck at math (no offense)
- Remember: *it's fine*, everyone does, no one's born a math whiz
- But it's a language you've got to learn
- And the only way into it is **practice**



Related courses at KU

- Contract theory, Auctions
- Economics of organization, Corporate Finance, Industrial organization
- Other courses in which applications of mechanism design appear:
 - Public finance/Taxation
 - Political economy
 - Monetary
 - Labor
 - ...

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Hi

- Egor Starkov
- Contact: `egor.starkov@econ.ku.dk` or absalon inbox
- Research interests: information economics, dynamic games, communication
- Office: 26.1.13
- Office hours: Tue, 14-15
- Questions: email/absalon, before/after class

Logistics

- Weekly lectures (except Fall break – week #42)
 - Tue, 15:15-18:00, CSS 35.0.12 (but check timetable for room changes)
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- Weekly problem sets (ungraded, for practice)
- Research module for PhD students: contact me if you would like to do it

Covid-specific stuff

- No online streams/recordings are currently planned, back to stone age

Workflow

- Intended workflow:

- 1 We start a topic in class
- 2 You watch video-lectures at home during the week (and/or read textbook, slides, papers)
- 3 We go through the same material *very quickly* in class and discuss any questions you have
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- Suggestion: organize into study groups. Watch videos in groups. Discuss problems in groups. Let me know by Friday if you want to join a group (assignment on absalon).
- In class I use whiteboard+slides for “lecture” stuff. Slides on absalon include some of the whiteboard parts.
 - I'll try to upload slides in advance, but they might be edited and updated afterwards

The problems are whiteboard-only; with solutions (mostly) uploaded afterwards.

References: textbooks

This course is a compilation of many books, papers, courses; does not follow any single one too closely. Below are some books that might help (see the reading list on Absalon for full references). Note that notation will be different across books and the class!

- **Narahari**: Probably your best bet. Hard to find in print, but you have online access through the library (see Absalon).
- **Diamantaras**: Another good textbook, but seems very hard to find.
- **Börger**: I used this as default in previous years, but it's quite hardcore and hard to follow. (Easy to find though.)
- **MWG**: A microeconomic bible. Very good, very clear, but has the smallest coverage for our course.
- **RS**: Relevant for two lectures on matching. Some material is contained in Narahari and Diamantaras. Nice reference if you are into matching.

References: other

- I will sometimes refer to individual papers and surveys for results outside of textbooks.
- Some of these are completely optional
- Some I expect you to know (but try to explain well enough in the slides).
- See the reading list on Absalon for details (will likely be updated during the course).

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First taste: Communication problem

Communication problem

Can *the designer* elicit *the agent's* **private information**? How?

- (“designer” and “agent” are just labels for different players for now)

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First taste: Communication problem

Communication problem

Can *the designer* elicit *the agent's* **private information**? How?

- (“designer” and “agent” are just labels for different players for now)
- As usual, **getting to an answer** first requires **understanding the problem**
- The answer depends on the **nature of the relationship**
 - What will the designer do with this knowledge?
 - How does the agent feel about that?
 - Let's look at a few examples...

Common interests

Example 1: Common interests

A tourist (principal) is asking a person on the street (agent) which way to the mermaid statue

(u_P, u_A)	$\theta = l$	$\theta = r$
$a = L$	2, 1	0, 0
$a = R$	0, 0	2, 1

- where $\theta \in \{r, l\}$ is the **state of the world** (the correct direction)
- $a \in \{L, R\}$ is the **action** the tourist will take (where they will go)
- the agent wants to help, knows θ

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- Let's consider a very sophisticated **mechanism** called “just ask for direction”.

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- $a \in \{L, R\}$ is the action the tourist will take (where they will go)
- the agent wants to help, knows θ
- Let's consider a very sophisticated mechanism called "just ask for direction".
- The agent would truthfully reveal θ , no reason to lie.

Opposed interests

Example 2: Opposed interests

A judge (principal) is asking a suspect (agent) whether they are guilty of a crime

(u_P, u_A)	$\theta = n$	$\theta = g$
$a = N$	1, 2	0, 2
$a = C$	0, 0	1, 0

- where $\theta \in \{g, n\}$ is the **type of the agent** (guilty or not)
- $a \in \{C, N\}$ is the judge's **action** (verdict: convict or not)
- the agent knows θ , wants $a = N$

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- $a \in \{C, N\}$ is the judge's action (verdict: convict or not)
- the agent knows θ , wants $a = N$
- Just asking would not work: if $\theta = g$, the agent would lie and say the state is $\theta = n$.
- It is not possible to elicit the agent's information
- (unless other tools are available)

Partially aligned interests

Example 3: Partially aligned interests

A president (principal) is asking an economic advisor (agent) by how much the spending should be cut.

- $\theta \in [0, 1]$ is the **state of the world** (optimal reduction)
- $a \in [0, 1]$ is the principal's **action**, wants $a = \theta - b$ for some bias $b < 0$
- the agent knows θ , wants $a = \theta$

Partially aligned interests

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- $a \in [0, 1]$ is the principal's action, wants $a = \theta - b$ for some bias $b < 0$
- the agent knows θ , wants $a = \theta$
- Crawford and Sobel [1982] show that if b small, there exists a **partially informative** equilibrium($[0, 1]$ is partitioned into intervals; agent reveals which interval θ belongs to)
- agent **reveals part** of their private information, but not all of it

Takeaways

- Agent is willing to reveal their private info if and only if it is in their interest to do so.
- Can we do better than “just asking”?

Takeaways

- Agent is willing to reveal their private info if and only if it is in their interest to do so.
- Can we do better than “just asking”? Turns out, **NO**.
- ...But if action is multi-dimensional, can often find partial alignment and **elicit at least some information**.

Partial alignment in multidimensional settings

Example 4: Multidimensional setting

A seller (principal) is asking a potential buyer (agent) about their valuation for the item.

- $\theta \in [0, 1]$ is the **type of the agent** (valuation)
- the principal's **action** consists of a price $p \geq 0$ and a decision $k \in \{s, r\}$ of whether sell or retain item
- the agent knows θ , wants $k = s$ if p low and $k = r$ otherwise
- the principal wants the opposite

Partial alignment in multidimensional settings

Example 4: Multidimensional setting

A seller (principal) is asking a potential buyer (agent) about their valuation for the item.

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- the principal's action consists of a price $p \geq 0$ and a decision $k \in \{s, r\}$ of whether sell or retain item
- the agent knows θ , wants $k = s$ if p low and $k = r$ otherwise
- the principal wants the opposite
- Despite opposing interests, trade **can** occur (e.g., via the seller announcing some price at which they're willing to sell, and the buyer choosing whether to accept it or not).

Takeaways

- Agent is willing to reveal their private info if and only if it is in their interest to do so.
- Can we do better than “just asking”? Turns out, NO.
- ...But if action is multi-dimensional, can often find partial alignment and elicit at least some information.

For next week

- 1 Split into study groups (if you are looking for a group: respond to an assignment on absalon by Friday or email me)
- 2 Have a look at math review notes on absalon
- 3 Watch lectures 2.1 ('What is a mechanism?') and 2.2 ('Dominant strategy implementation') on youtube.
- 4 Or read Narahari ch.14-16

References I

V. P. Crawford and J. Sobel. Strategic Information Transmission. *Econometrica*, 50(6): 1431–1451, 1982. URL <https://doi.org/10.2307/1913390>.