Exercises for Lecture 2 (M1): Revelation Principle, DSIC Mechanisms

Problem 1: Søndre campus

There are currently talks at KU about moving the Faculty of Social Sciences from the Kommunehospitalet that we occupy now to Søndre campus, where some other faculties are currently located.[1] The costs and benefits of such a move are currently being evaluated. Some, however, see this whole discussion as a bargaining maneuver in the upcoming negotiations with the firm that owns the Kommunehospitalet and leases it to the university – a credible threat of leaving may help the university bargain a better lease rate.

Your mission is to frame this choice of whether SAMF should move as a mechanism design problem. The goal of the mechanism is to extract the information about costs and benefits of the potential move from the relevant parties. In particular, answer the following questions within this setting:

1. Who is the designer?
2. What is the outcome in this setting? (Do we have access to transfers? Is the set of allocations k given by simply $K = \{\text{move}, \text{no move}\}$ or is it more multifaceted?)
3. Who are the players?
4. What information do the players have that is relevant to determining the optimal outcome/allocation?
5. How would you model the players' utility functions? (Give a concrete example.)
6. What criteria or conditions should the mechanism satisfy?
7. What would be the desirable outcome/allocation rule that you would want to implement with such a mechanism? How can you check whether this rule is, in fact, implementable?
8. If you allowed for transfers: how would you proceed with designing transfers that support the chosen allocation rule? (You do not need to actually derive the transfers.)
9. How would your mechanism work in the real world, in terms of organization and logistics?[2]

NOTE: treat this as a real-life assignment from the university officials. Your goal is to give the best possible answer to the question they ask, NOT to frame the problem in the simplest way possible. That said, you should still be realistic and try to set up the problem in a way that would be tractable and doable given the resources available to a committee responsible for this decision.

Problem 2: Screening

One application of mechanism design is to profit maximization when consumers have private information about their valuations. One example of such a problem is second-degree price discrimination that you have seen in mikro II. The following is a variation of that, known as a "monopolistic screening" problem (with two types) that you may have seen in game theory.

Suppose a seller-designer offers a single product for sale that he can produce at zero cost. She offers a menu of pairs of quantities $k \in [0, 1]$ and payments $t \in \mathbb{R}_+$ (for the whole amount k, not per unit). There is one buyer with valuation $\theta \in \{L, H\}$ for the product, which is his private information. The seller's belief

[1] News article from Uniavisen (in Danish): `https://tinyurl.com/y4uwrefe`.

[2] Example: "all faculty, staff, and students must post a note on the door of their office which would contain their `report of something`; a dedicated person will walk around and enter responses in an excel sheet, which will then be used to determine the outcome".

regarding θ is given by $\phi(H) = \phi$, $\phi(L) = 1 - \phi$. The buyer's preferences are given by $u_b(k, t, \theta) = \theta k - t$ if he buys the product and zero otherwise.

1. Explain why it is sufficient for the seller to offer a menu consisting of two items: (k_H, t_H) and (k_L, t_L).
2. Write down the seller's problem of maximizing her expected profit subject to the buyer's incentive compatibility (IC) and individual rationality (IR) constraints for every θ, in terms of the model primitives.
3. Derive the seller's optimal menu $((k_H^*, t_H^*), (k_L^*, t_L^*))$ by following the steps below.
   (a) Show that if $((k_H, t_H), (k_L, t_L))$ satisfy IC_H and IR_L, then they also satisfy IR_H.
   (b) Show that $((k_H, t_H), (k_L, t_L))$ satisfy IC_H and IC_L only if $k_H \geq k_L$.
   (c) Show that if $((k_H, t_H), (k_L, t_L))$ are such that $k_H \geq k_L$ and IC_H binds (i.e., is satisfied with equality), then they also satisfy IC_L.
   (d) Show that given all of the above, it is always optimal to choose $((k_H, t_H), (k_L, t_L))$ in such a way that IC_H and IR_L bind.
   (e) Given all of the above, solve for the optimal menu $((k_H^*, t_H^*), (k_L^*, t_L^*))$.

Problem 3: Screening 2

This is a marginally more difficult version of the previous problem. Once you understood the solution of the previous problem, try to solve this one by following the same algorithm.

The Chicago Transit Authority (the organization in charge of the Chicago subway system) has decided that it needs to do more to maximize its revenue. As such it has hired you to design its new price and service scheme. There are two types of customers, High-class and Low-class. They have preferences over the fare P and the degree of bad smell in the train car they ride in, denoted by B. They have told you that they are able to charge different fares depending on the car a customer rides in (i.e., to have different classes of service).

The type of a customer is not observable; the fraction of high-class customers is λ. Customers' utility functions are $u_i(P, B) = v - \theta_i P - B$, for $i = H, L$, where $\theta_L > \theta_H > 0$. All customers get utility (normalized) of 0 from walking (their next best alternative) instead of taking the CTA train.

Making train cars smell bad is not costless (workers need to be hired to rub garbage on the seats): the CTA incurs a cost of $\gamma B > 0$ per customer who rides in a car that has smell level B.

1. Write down the problem you would solve for determining the CTA's profit-maximizing scheme. Assume throughout that the CTA cannot charge negative prices; i.e., that $P \geq 0$. Assume also that the CTA wants to serve both high and low class customers.
2. Determine the CTA's profit-maximizing scheme. How does it depend on the parameters of the problem?

Problem 4: Judicial design

A suspect is in custody, accused of murder. If he goes to trial he will either be convicted or acquitted. If he is convicted he will be sent to prison for life giving him a payoff of -1. If he is acquitted he goes free and has a payoff of 0. The district attorney can offer plea bargains: allowing the defendant to plead guilty in return for a lighter sentence. In particular, for any $r \in (0, 1)$, the DA can offer a reduced sentence which, if accepted, would give the defendant a payoff of $-r$.

The defendant is privately informed about his chances for acquittal at trial: $\theta \in [0, 1]$ is the defendant's privately known probability of acquittal. If the defendant does not enter into a plea bargain with the DA he will go to trial and be convicted with probability $1-\theta$.

Consider the mechanism design problem where the DA is the principal and the defendant is the agent. A social choice function is a mapping $f:[0,1]\to\{\text{trial}\}\cup(0,1)$ where $f(\theta)=\text{trial}$ means that type θ will go to trial and $f(\theta)=r\in(0,1)$ means that type θ accepts a plea bargain giving him a sentence with payoff $-r$. DA thinks θ has full support on $[0,1]$.

1. Write down the inequalities that characterize whether some given social choice function f is incentive-compatible for the defendant.
2. What is the set of all incentive-compatible social choice functions? You can proceed in the following steps:
   - Show that in any IC f at most one plea bargain r is available.
   - Show that f must be of cutoff type, with the suspect taking the plea if $\theta<\bar{\theta}$ and going to court otherwise.
   - Find the value of r that makes the cutoff s.c.f. f incentive compatible given some cutoff type $\bar{\theta}$.
   - Combine all of the above to characterize the set of implementable f.

Problem 5: Second-price auction

There is a single item being sold via a second-price sealed bid auction. There are $i=1,\dots,n$ bidders. Every bidder i has a private valuation θ_i, which the other players believe is distributed according to some c.d.f. $F_i(\theta_i)$. All bidders simultaneously submit bids b_i to the seller (without seeing what the others bid). The highest bidder then wins the object and pays the second-highest bid $b_{(2)}$, their utility is then given by $u_i=\theta_i-b_{(2)}$. All other bidders get nothing and pay nothing, so their utility is zero.

1. Show that bidding truthfully ($b_i=\theta_i$) is a weakly dominant strategy for every bidder i.
2. Conclude that the second-price auction implements the efficient allocation rule in dominant strategies.