Exercises for Lecture 7 (M4): Correlated information.

Problem 1: Meeting the investor

A team of two entrepreneurs $i = 1, 2$ approaches a venture investor $i = 0$ with a request to fund their newest business idea. Suppose the real value of the idea is $\omega \sim U[-\infty, \infty]$.[1] Each entrepreneur $i = 1, 2$ estimates this value at $\theta_i = \omega + \epsilon_i$, where $\epsilon_i \sim i.i.d.U[-0.5, 0.5]$. (Take unit of measurement to be millions of dollars.)

1. Design a mechanism that would allow the investor to perfectly learn both entrepreneurs' estimates θ_i. In this mechanism, both would independently report their θ_i to the mechanism, and the mechanism would prescribe report-contingent transfers $t_i(\theta_1, \theta_2)$ from each entrepreneur $i = 1, 2$ to the investor. Assume that the two entrepreneurs cannot coordinate their reporting strategies. Derive the transfer rules that implement truthful reporting (and show that it is indeed optimal for both entrepreneurs to report truthfully under this transfer rule).
2. How could your mechanism be implemented in the real world? I.e., is it reasonable to ask entrepreneurs to pay for a meeting with an investor? If not, how else could you induce the desired transfers?

Problem 2: Optimal Auction with Correlated Values

Consider the optimal auction problem in which there are two bidders whose valuations $v_i \in [0, 1]$ are uniformly distributed but perfectly correlated, i.e. $v_1 = v_2$ with probability 1. Construct a DSIC mechanism with the following properties:

1. Bidder 2 wins the object regardless of the type profile.
2. Both bidders earn zero utility at every type profile.

Problem 3: Cremer-McLean

There are two players, $i = 1, 2$. Each of them has one of two types, $\theta_i \in \{H, L\}$. The joint distribution of types is given by $\phi(\theta_1, \theta_2)$ as follows:

	H	L
H	$\frac{1}{6}$	$\frac{1}{3}$
L	$\frac{1}{3}$	$\frac{1}{6}$

Both players have quasilinear utilities.

First explore the problem of information elicitation (without the need to support any underlying allocation).

1. Compute the players' interim beliefs $\phi(\theta_{-i}|\theta_i)$.
2. Compute the truth-revealing transfers $\hat{t}_i(\theta) = -\ln(\phi(\theta_{-i}|\theta_i))$.
3. Verify that a direct mechanism in which each player reports their type θ_i and pays $\hat{t}_i(\theta)$ is BIC (when coupled with some constant allocation rule k).

Now suppose that the society of these two individuals chooses whether to adopt a new bank holiday called "National Equality Day", so the "real outcome" is $k \in \{1, 0\}$ (where 1 means bank holiday and 0 means

[1] This is called an "improper prior" – the prior belief about ω is not a proper probability distribution, but the posterior belief resulting from updating it via Bayes' rule after an informative event would be a proper distribution.

none). The holiday should only be adopted if all citizens are, in fact, equal, i.e. the desired allocation is $\tilde{k}(\theta) = \mathbb{I}\{\theta_1 = \theta_2\}$. Each citizen receives utility 1 if the holiday is adopted and 0 otherwise.

4. Is $\tilde{k}$ efficient?
5. Show that $\tilde{k}$ cannot be sustained without transfers, i.e. that a mechanism $(\tilde{k}, t)$ with $t(\theta) = 0$ for all θ is not BIC.
6. Consider transfers $t_i(\theta_i, \theta_{-i}) = C_1 + C_2\hat{t}(\theta_i, \theta_{-i})$. Derive conditions on values of C_1, C_2 for which a direct mechanism $(\tilde{k}, t)$ is BIC.
7. Derive conditions on values of C_1, C_2 for which a direct mechanism $(\tilde{k}, t)$ is interim IR and ex ante BB.
8. Give an example of a BIC, interim IR and ex ante BB mechanism that implements $\tilde{k}$.