Financial Markets Microstructure: Problem Set 1

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Problem 1

Exercise 4 in chapter 3 (pp. 125-126):

Consider the one-period Glosten-Milgrom model with $v \in \{v^L, v^H\}$ and $\mathbb{P}(v^H) = 1/2$. The dealer is competitive and risk neutral, so prices are equal to expected values. With probability $1 - \pi$ the trader is a noise trader who buys and sells with equal probability, with probability π he is a potential insider. The potential insider can, before submitting a trade, privately acquire information that perfectly reveals v at some fixed cost $c > 0$. If no information is acquired, the potential insider has no private information about v. As usual, suppose the dealer cannot distinguish the traders.

(a). Suppose an insider decided to not acquire any information. What trading strategy maximizes his expected profit?

(b). Compute the bid and ask prices set by the dealer, assuming that he believes that the insider acquires information with probability $\varphi \in [0, 1]$.

(c). Given these prices, determine the trading profit of an insider who has decided to acquire information.

(d). There exists $\bar{c}$ such that if $c > \bar{c}$, then the potential insider never acquires information (i.e., in equilibrium, $\varphi = 0$). Calculate $\bar{c}$.

(e). There exists $\underline{c}$ such that if $c < \underline{c}$, then the potential insider always acquires information (i.e., in equilibrium, $\varphi = 1$). Calculate $\underline{c}$.

(f). Suppose now that $c \in (\underline{c}, \bar{c})$. Find the mixed-strategy equilibrium in which the potential insider learns v with positive probability smaller than one. Describe the equilibrium fully.

Problem 2

This problem considers an issue left open in Section 4.3 in the textbook, which explores inventory risk within the Kyle's model. At the outset of this section, it is assumed that there is no informed trading, i.e., $x = 0$ and $q = u$. Let us now relax this assumption and try to analyze the effects of dealer's risk aversion in Kyle's model in the presence of adverse selection.

So, consider a combination of the two versions of the Kyle model we have seen in class: we now have one informed trader, a pool of noise traders, and a representative competitive risk-averse dealer with mean-variance preferences, risk aversion ρ, initial asset holding z_0, and initial cash holding c_0. I.e., the dealer's utility from a [random] next period's wealth $w_{t+1} = z_{t+1}p_{t+1} + c_{t+1}$ is $U(w_{t+1}) = \mathbb{E}[w_{t+1}] - \frac{\rho}{2}\mathbb{V}(w_{t+1})$.

We are again looking for a linear equilibrium, where the insider submits an order $x = \beta v - x_0$, and the pricing schedule quoted by the dealer is $p = p_0 + \lambda q$. Now, there are four endogenous parameters, β, x_0, p_0, λ, with $\beta > 0$ and $\lambda > 0$.

(a). Take for granted some arbitrary parameters β, x_0. The dealer observes $q = x + u$. Use the result from Section 4.2 or lecture slides to find $\mathbb{E}[v|q]$ and $\mathbb{V}[v|q]$ as functions of β and the model parameters.

(Note that it is no longer the case that $q = \beta(v-\mu)+u$, but rather $q+x_0-\beta\mu = \beta(v-\mu)+u$.)

(b). The competitive dealer takes p as given. In equilibrium, $p = p_0 + \lambda q$, so the price reveals q – same logic as in Section 4.2.4. Rewrite $\mathbb{E}[v|q]$ and $\mathbb{V}[v|q]$ from (a) as functions of p instead of q, when $q = (p - p_0)/\lambda$. Let's call these functions $\mathbb{E}[v|p]$ and $\mathbb{V}[v|p]$.

(c). Calculate the dealer's asset supply curve and show that the amount supplied at price p is

$$y(p) = z_0 + \frac{p - \mathbb{E}[v|p]}{\rho\mathbb{V}[v|p]}$$

(A reminder: To solve this problem, instead of the zero-profit condition you should invoke another feature of competitive agents, namely price-taking. Use the following algorithm to obtain the dealer's price schedule: (1) fix some arbitrary price p; (2) find supply amount $y(p)$ that maximizes the dealer's profit given this p; (3) invert the supply function $y(p)$ to obtain a pricing schedule $p(q)$.)

(d). When the market clears at price p, then $y(p) = q$. Can you see if there exists a $\lambda > 0$ and a number p_0, such that market clearing fits with our conjecture that $p = p_0 + \lambda q$? I.e. try to solve the following equation

$$(y(p) =) \quad z_0 + \frac{p - \mathbb{E}[v|p]}{\rho\mathbb{V}[v|p]} = \frac{p - p_0}{\lambda} \quad (= q)$$

w.r.t. λ and p_0. Note that they must be such that the equality holds for all p.

(e). Go back to the informed trader's problem. Assuming that $p = p_0 + \lambda q$, argue that the insider chooses x to maximize $x(v - p_0 - \lambda x)$. Show that the relation is of the form $x = \beta v - x_0$ and try to relate parameters β, λ, x_0, p_0 to each other. Is there a solution for all four parameters? If the algebra is impossible, try a numerical solution.

(f). In case $\rho = 0$, the dealer is risk neutral, so we are back to the case of pure adverse selection. In this limit case, do you get back the equilibrium found in Section 4.2/in class? How do you find that changes in ρ affect market depth in equilibrium?

Problem 3

One of the news articles assigned earlier in the course argued that corporate bond markets are significantly less liquid than stock markets. This is partly due to a more opaque trading tradition: instead of the dealers posting their quotes openly, the bond markets operate via Requests For Quotes (RFQ). In particular, an interested trader must contact the dealer with an RFQ, and then the dealer would respond with a quote. This process is somewhat time-consuming for traders, and thus costly. However, the costs of this opaqueness can be far greater than just these time costs.

Your goal is to build a model of search costs and to examine how they affect market outcomes: prices, spread and/or market depth, volume of trade if applicable. In particular, take any dealer model we considered (I suggest Glosten-Milgrom, but Kyle can probably work as well). Assume that instead of the dealers posting quotes openly, any arriving trader must approach dealers individually, paying cost c per dealer to learn that dealer's quotes. Solve your model as best you can and answer the following questions based on your results:

- How do prices (and thus liquidity/depth) in your model compare to the baseline model, in which dealers post quotes openly?
- How do prices depend on c?
- How do prices depend on the number of dealers in the market?
- In this situation, could a dealer profit by announcing quotes publicly?