

Financial Markets Microstructure

Lecture 18

Asset Price Bubbles

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Previously on FMM

- Information and trading volumes:
 - Private signals create disagreement → generate trade, which reveals and aggregates private info
 - Public signals should theoretically mitigate disagreement and lead to less trade
 - But IRL trading volumes increase around public announcements
 - **Kondor**: Possible explanation through second-order beliefs

Today

Two stories for why **bubbles** may occur

- Rational herding
 - Herding: following the actions of others, even when this goes against one's own private information
 - We will look at different explanations for why this may be rational
- Lack of common knowledge/coordination (Abreu and Brunnermeier [2003])
 - There is a difference between everybody knowing that an asset is overpriced, and everybody knowing that everybody knows...
 - Again, speculation depends on these *higher-order beliefs*

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Bubbles

- **Wikipedia:** “Trade in high volumes at prices that are considerably at variance with intrinsic values”
- **Investopedia:** “A surge in equity prices, often more than warranted by the fundamentals and usually in a particular sector, followed by a drastic drop in prices as a massive sell-off occurs”
- **Chicago Fed:** “...a bubble exists when the market price of an asset exceeds its price determined by fundamental factors by a significant amount for a prolonged period”

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Herding: Introduction

- **Herding**: ignoring private information in favor of “wisdom of the crowd”
 - 1 Herding may be the efficient response to new information or similarity between agents...
 - 2 ...but it may also be the inefficient result of a certain decision making process. Here: focus on the latter
- In **financial markets**: momentum trading, positive-feedback trading
- In 1992, a wave (a herd?) of articles showing that herding could be result of rational [informational cascade](#)
- Today we look at this and other ‘rational’ explanations
- Note: I will follow the presentation in Bikhchandani and Sharma [2000].
See Bikhchandani et al. [2024] for a comprehensive review of the literature.

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Preview

- Agents arrive at the market sequentially and need to make a decision
- Every agent has a private signal and observes decisions of previous agents (but not their signals!)
- Ideally: pool private information to find best decision
- But here: [sequential decision making](#)
 - First-comers: make decisions based on information
 - But as time goes by, people may start disregarding their own information and just choose the most popular action
 - Herding ensues, but it is *fully rational*: public information swamps private information
- [Informational cascade](#): a few pieces of information may determine everyone’s choice

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Model

- **State** (fundamental) $v \in \{L, H\}$;
- In each period $t \in \{1, 2, \dots\}$ an individual arrives and needs to make a **decision** $d_t \in \{0, 1\}$ (invest or not);
- **Payoffs** $u(d_t, v)$:
 - $u(0, v) = 0$,
 - $u(1, L) = L - m < 0$,
 - $u(1, H) = H - m > 0$,
 - (say for now that price (midquote) m is fixed and there is no spread: $a = b = m$)

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Model: Beliefs

- Represent all beliefs p about v as probabilities of $v = H$ conditional on relevant info.
- “Market belief” q_t
 - q_0 is the public prior belief – e.g., $1/2$;
 - q_t incorporates info contained in decisions at $t = 1, 2, \dots, t - 1$
- Period- t agent observes q_t and a private signal η_t and forms private belief r_t .
 - Suppose $\eta_t \in \{h, l\}$ with $\mathbb{P}(\eta_t = h | v = H) = \mathbb{P}(\eta_t = l | v = L) = \rho$.
- All beliefs calculated using Bayes' rule

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Decisions

- Agent behaves optimally \Rightarrow chooses $d_t = 1$ iff $r_t \geq \bar{r} \equiv \frac{m-L}{H-L}$.
- Use Bayes' rule to compute $r_t(\eta_t, q_t)$:

$$r_t(h, q_t) = \frac{q_t \rho}{q_t \rho + (1 - q_t)(1 - \rho)}$$
$$r_t(l, q_t) = \frac{q_t(1 - \rho)}{q_t(1 - \rho) + (1 - q_t)\rho}$$

Note $r_t(h, q_t) > q_t > r_t(l, q_t)$.

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Herds and cascades

- If $r_t(h, q_t) > \bar{r} > r_t(l, q_t)$ then the agent follows their signal
 - \rightarrow their action is informative
 - \rightarrow public belief is updated: $q_{t+1}(q_t, d_t = 1) = r_t(h, q_t)$; $q_{t+1}(q_t, d_t = 0) = r_t(l, q_t)$.
- If $r_t(h, q_t) > r_t(l, q_t) > \bar{r}$ then the agent chooses $d_t = 1$ regardless of private signal
 - \rightarrow action uninformative $\rightarrow q_{t+1} = q_t$
 - \rightarrow next agent will also ignore private signal!
 - Everyone ignores private signals and chooses the same action (we have a **herd**)
 - Market belief q_t is frozen in place; private information is not aggregated!
 - This **herd may be incorrect** (unless $q_t = 0$ or $q_t = 1$)
 - (Same happens if $\bar{r} > r_t(h, q_t) > r_t(l, q_t)$ with $d_t = 0$)

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What triggers the herd?

- A few incorrect signals can be enough to set off a herd
- A small amount of information 'cascades' through the system
- Each agent is rational, but together they may seem stupid: their information taken together is very precise, but information aggregation fails
- A specific example with numbers of how a herd may arise: [here](#)

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Herds and cascades: comments

- Incorrect herds only occur if distribution of η_i is bounded – otherwise strong enough signals could overpower the public info
- In a slightly richer model, the opposite outcome is also possible – state of permanent uncertainty in which everyone acts solely on their private signal, ignoring public information
- Terminology:
 - **Herd** = action convergence ($d_{t+1} = d_t$ from some point onwards)
 - **Cascade** = public belief convergence ($q_{t+1} = q_t$ from some point onwards)
 - Distinction is not super important for our purposes

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What if we introduce a price?

- As before: unknown asset value v , and each trader receives a private signal η_i
- With probability π the trader is a noise trader, who buys/sells/abstains with equal probability, w.p. $1 - \pi$ rational as above
- A risk-neutral market maker quotes competitive bid-ask **prices**
- What will happen?
 - This is standard Glosten-Milgrom, so no herds!
 - Prices adjust in such a way that following private signals is optimal, and prices themselves then incorporate all private signals!

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More layered model

- However, the GM result is in part due to model simplicity
- Avery and Zemsky [1998] expand on this analysis
 - Suppose $v \in \{0, \frac{1}{2}, 1\}$
 - If $v = \frac{1}{2}$, this is perfectly revealed by the private signal: $\eta_t = \frac{1}{2}$
 - If $v \in \{0, 1\}$, trader t receives informative signal with precision $\rho_t \equiv \mathbb{P}(\eta_t = v)$
 - Furthermore, a proportion $\mu \in \{\mu^H, \mu^L\}$ of traders have perfect information: $\rho_t = 1$
 - The remaining $1 - \mu$ traders have noisy info: $\rho_t \in (\frac{1}{2}, 1)$
- Three levels of uncertainty:
 - Event uncertainty: $v = 1/2$ (**no event**) or $v \in \{0, 1\}$ (**event**)
 - Value uncertainty: if event, $v = 1$ or $v = 0$
 - Composition uncertainty: many informed traders (μ^H : **well-informed economy**) or few (μ^L : **poorly-informed economy**)

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Herding can occur with pricing and multi-layered uncertainty

- In GM, price mechanism worked as a screening device: made sure that high types bought and low types sold
- But now, it is possible to 'misprice' such that herding occurs, at least temporarily
 - Can be because $\mu = \mu_H$ so all traders know v , but MM does not (non-speculative bubble)
 - Can be because $\mu = \mu_L$ but traders do not know that and perceive past order flow as more informative than it is (speculative bubble)
- This allows bubbles to occur

See numerical example [here](#)

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Other types of herding: Reputational herding

A brief example.

- **Two managers:** Invest or not in project of unknown value (gets imperfect signal $\eta \in \{1, 0\}$)
- **Type:** Each manager either smart or dumb (doesn't know). If both managers smart, observe same η ; if one or both dumb, observe independent signals η
- **Payoffs:** Managers [maximizes their reputation](#) (want to appear smart)
- **Herding:** Manager 1 moves first, then manager 2
 - If manager 1 invests, manager 2 can deduce that $\eta_1 = 1$. Suppose $\eta_2 = 0$. Then one of the two (or both) must be dumb.
 - If manager 2 does not invest, he reveals this. If player 1's investment then succeeds, people will assume that manager 2 is the dumb manager
 - Investing might be better: even if the investment fails, people might think that both managers are smart, but got an 'unlucky' signal

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Herding: Conclusion

- **Herding**: may occur when private information cannot be easily aggregated
- **Price mechanism**: alleviates problem by providing private incentives to trade and thus reveal information, which is then incorporated into prices
- **Multi-layered uncertainty**: can make the herds occur even with flexible prices
- **Aside**: *Momentum trading* often assumed to be a 'behavioral feature': but it may be perfectly rational
- **Empirical estimation of herding**: some conclusions can be tested in the data but to a very limited extent; see Bikhchandani and Sharma [2000] and Bikhchandani et al. [2024] for details.

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Abreu and Brunnermeier [2003]: Introduction

- Efficient market hypothesis: bubbles are impossible
 - In finite model, use backward induction argument: no mispricing in last period, which should be foreseen in second-last period, etc. → unravelling
- This requires that all traders agree on when the bubble will collapse (at least on the distribution of times)
- Here: a model in which coordination is needed to cause a crash
- Coordination, in turn, depends on beliefs about others
 - Back to higher order beliefs

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Model

- **Single asset traded:** value at t is v_t
- **Progress:** At $t = 0$, technological progress makes v_t grow at rate g
- **Slowdown:** at some random time t_0 , there is a slowdown and v_t growth slows to $r < g$
- **Price:** Price grows at rate g until either
 - At least a fraction κ of rational traders sell the asset (κ is the *absorption capacity* of the economy), or
 - The market is exogenously corrected at time $t_0 + \bar{\tau}$
- **Gradual learning:** Each period, a fraction $1/\eta$ of rational traders become aware of the mispricing
 - But they don't know t_0 , and hence don't know how many others know of the mispricing

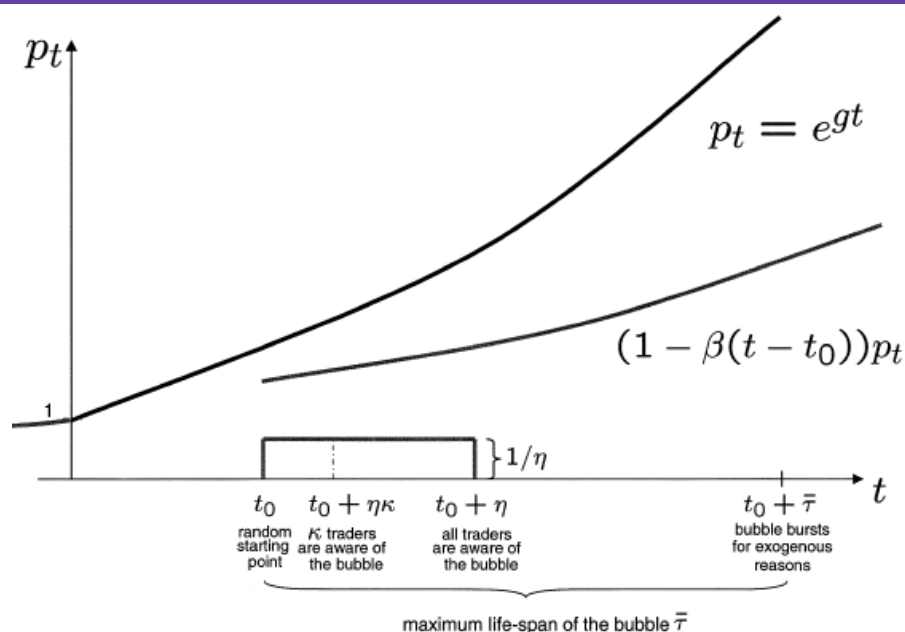
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Traders

- **Behavioral traders** think progress will last forever
 - When enough rational traders sell, absorption capacity of behavioral traders is reached and price must drop
 - At this point, everybody becomes aware of the mispricing
 - If this doesn't happen, then market exogenously collapses when mispricing is too big
- **Rational traders** know progress is temporary, but not when it stops
 - Implication: when you learned that progress has stopped, you are not sure what other people believe
 - Suppose you learn at t' . At $t' + \eta$ you know that everybody knows
 - But somebody else might learn that progress stopped at $t'' > t'$; he will not know that everybody knows until $t'' + \eta > t' + \eta$

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Graphically



(Note: graph is from the paper and it has a mistake)

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Why bubbles then?

- In this model there is never *common knowledge*: you are never sure what the others know. This prevents usual backward induction
- At least κ traders must sell to burst bubble: coordination important
- But it is hard to coordinate without common knowledge
- Therefore, mispricing can go on for a long time

Even if traders realize the market will crash, they don't know exactly when, incentivizing them to ride the bubble

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Definition of a bubble

- In this setting, let's use a very strict definition of a bubble:
mispricing should be well-known to traders to count as a bubble

Definition

A bubble is persistent mispricing *beyond* $t_0 + \eta\kappa$

- After $t_0 + \eta\kappa$, the mispricing is known by enough traders to correct it, should they want to

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Some observations

The following is shown in the paper:

- Traders either take the maximum long (buy) or short (sell) position
- When a rational trader 'goes short', all traders who learned of the mispricing before him will already have gone short
- Once a rational trader goes short, he never re-enters the market: he waits for the bubble to burst

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Bubbles and crashes

Generally, in the model, there are two possible types of equilibria

- Exogenous crash
 - When the growth rate g under the bubble is high, dispersion η and absorption capacity κ are high, informed traders sell out very slowly
 - In effect, they take a chance on 'riding the bubble'
 - As a result, selling will never be sufficient to burst the bubble, which will burst at the exogenous date $t_0 + \bar{\tau}$
- Endogenous crash
 - When traders have incentives to sell quick, this leads to unraveling: enough traders will sell to make the bubble burst
 - However, 'bubble incentives' remain: the sooner the price crashes, the smaller the cost of riding the bubble
 - Therefore, the bubble will be smaller but still exist

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The role of 'sunspots'

- **Sunspot equilibria:** refers to equilibria where *economically irrelevant information* has an influence
- In our case: suppose an uninformative event is observed with a certain probability
- If informed traders decide to coordinate their actions around this event (eg. use the strategy 'sell when event occurs'), it can become pivotal
- Example from article: in 1980s trade data had big market impact in the US; in 1990s Alan Greenspan's statements were more influential
 - If sufficiently many people react to an event, then you must too - even if the event carries little/no information by itself

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Abreu and Brunnermeier: Conclusion

- Standard arguments against bubbles rely a lot on common knowledge between agents
- When we dispense with common knowledge, belief dispersion among agents can cause mispricings to persist even after everyone has observed the mispricing
- Thus, bubbles may not be fixed by the market
- As a side-effect, seemingly insignificant events can serve to coordinate actions and cause crashes

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Herding example: Setup

- This example follows that in Bikhchandani and Sharma [2000]
- Suppose $q_0 = 1/2$
- **Payoffs:** Let $L - m = -1$; $H - m = 1$
 - Then $\bar{r} = 1/2$
 - Suppose that agents flip a 50/50 coin whenever indifferent
- Suppose the following signal sequence realized: $\{h, l, l, l, \dots\}$
- Denote by \mathbb{I}_i agent i 's information set

Herding example: Analysis

First agent

- Information set $\mathbb{I}_1 = \{\eta_1 = h\}$

- Then

$$r_1(h, 1/2) = \mathbb{P}(v = H | \mathbb{I}_1) = \frac{\frac{1}{2}\rho}{\frac{1}{2}\rho + \frac{1}{2}(1-\rho)} = \rho$$

- Since $\rho > 1/2$: invest.
- **Thus**, given $\eta_1 = h$, first agent invests: $d_1 = 1$

Herding example: Analysis (2)

Second agent

- Second agent can perfectly deduce first agent's signal: $\eta_1 = h$ if $d_1 = 1$, $\eta_1 = l$ otherwise.
 - In other words, $q_2 = r_1 = \rho$, no information is lost.
 - Second agent receives $\eta_2 = l$, his information set is $\mathbb{I}_2 = \{\eta_1 = 1, \eta_2 = -1\}$

- Signals are *symmetric*, so

$$r_2(l, \rho) = \mathbb{P}(v = H | \mathbb{I}_2) = \frac{\frac{1}{2}\rho(1-\rho)}{\frac{1}{2}\rho(1-\rho) + \frac{1}{2}(1-\rho)\rho} = 1/2$$

- Thus, second agent is indifferent, flips a coin to decide. If $d_2 = 0$ then we are back to $q_3 = 1/2$.
- But suppose the coin-toss decides invest: $d_2 = 1$

Herding example: Analysis (3)

Third agent

- Third agent can also perfectly deduce the first agent's signal
- Information set is $\mathbb{I}_3 = \{\eta_1 = h, d_2 = 1, \eta_3 = l\}$. Furthermore,

$$\mathbb{P}(d_2 = 1 | v = H) = \rho + (1 - \rho)(1/2) = (1 + \rho)/2$$

$$\mathbb{P}(d_2 = 1 | v = L) = \rho(1/2) + (1 - \rho) = 1 - \rho/2$$

$$\Rightarrow q_3(q_2 = \rho, d_2 = 1) = \frac{\rho^{\frac{1+\rho}{2}}}{\rho^{\frac{1+\rho}{2}} + (1 - \rho) \left(1 - \frac{\rho}{2}\right)}$$

$$\Rightarrow r_3(l, q_3) = \frac{\rho^{\frac{1+\rho}{2}}(1 - \rho)}{\rho^{\frac{1+\rho}{2}}(1 - \rho) + (1 - \rho) \left(1 - \frac{\rho}{2}\right) \rho} = \frac{1 + \rho}{3} > \frac{1}{2}$$

- Hence, $d_3 = 1$

Herding example: Analysis (4)

Fourth agent

- Information set $\mathbb{I}_4 = \{\eta_1 = h, d_2 = d_3 = 1, \eta_4 = l\}$
- Agent four knows that agent three would have invested regardless of his signal
- So he doesn't learn anything from d_3 . So $q_4 = q_3$ and $r_4(l, q_4) = r_3(l, q_3)$
- Hence $d_4 = 1$
- Same for all subsequent agents: $d_i = 1$ for all i , regardless of η_i !
- We have a herd! – and a very inefficient one at that! (Look at signals)

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Herding with prices example

Take an extreme case of Avery and Zemsky's model: suppose the following parameter values

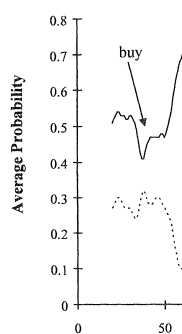
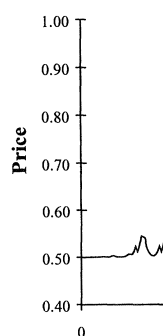
- $\mathbb{P}(v = 1/2) = 0.9999$: very small prior probability of an 'event';
 $\mathbb{P}(v = 1) = \mathbb{P}(v = 0) = 0.00005$
- $\frac{\mathbb{P}(\mu = \mu^H)}{\mathbb{P}(\mu = \mu^L)} = 99$: high prior probability of a well-informed economy
- If economy is poorly informed: all traders have $\rho_i = 0.51$, i.e. very poor signal about value

Suppose we're in an unlikely state of the world

- 1 Value is low ($v = 0$) implying that there is an event
- 2 The economy is poorly informed ($\mu = \mu^L$)

How will market learn state of the world? Let's look at a simulation

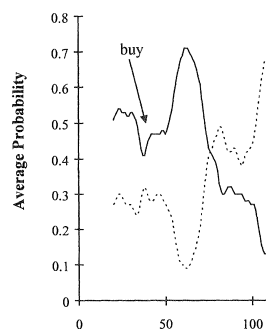
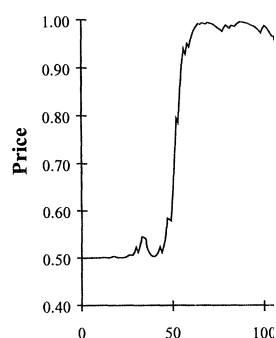
Herding with prices example (2)



Three of first five traders buy

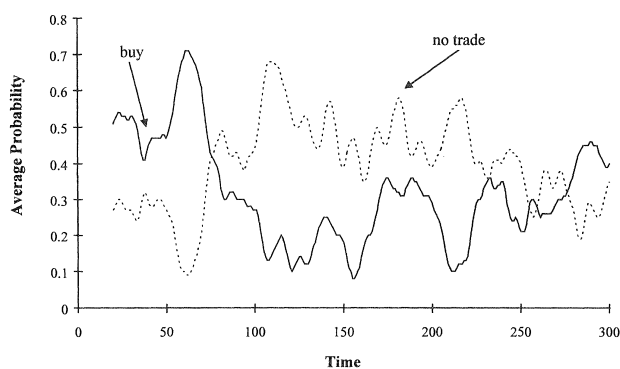
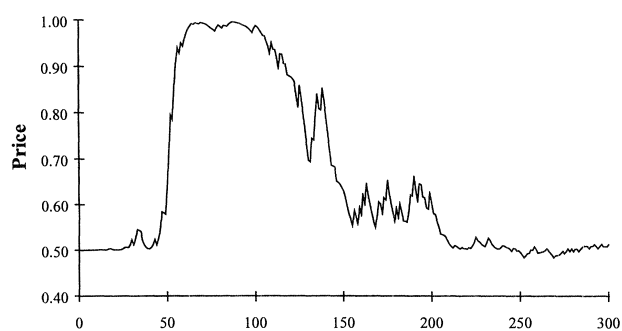
- Since economy poorly informed: herd buying starts
- MM thinks it's likely that economy is well informed: price goes up

Herding with prices example (3)



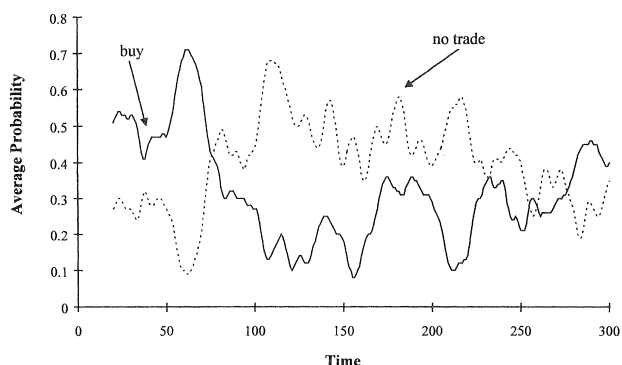
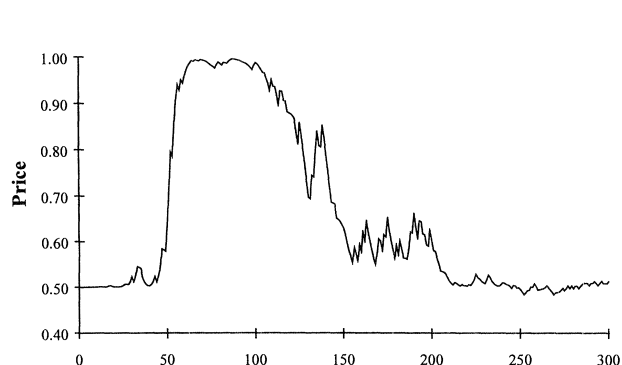
As the price goes up, the herd is broken: trading volume diminishes as rational traders drop out

Herding with prices example (4)



As herd becomes apparent, MM realizes that only a few informative trades have been made → price toward $1/2$

Herding with prices example (5)



Rational traders only re-enter in period 220: trade based on information (buy if high/sell if low)

In the example, the price is persistently above the level that would ensue if traders pooled their information

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