This lecture:

1. Glosten-Milgrom model
   - ex 3
   - GM with Uniform value

- FPR chapter 3 exercises (p. 124-125):
  - exercise 3 about the GM model where speculators are not perfectly informed, but instead receive a signal about the value of the asset
- Bonus problem: GM model with uniform v

Exercise 3

- $v \in \{v^H, v^L\}$ w.p. $\frac{1}{2}$;
- dealer competitive, risk-neutral, does not know v;
- trader uninformed w.p. $1 - \pi$, then buys and sells w.p. $\frac{1}{2}$;
- trader informed w.p. π, then observes signal about state
  - signal accurate w.p. $\rho \in \left(\frac{1}{2}, 1\right]$.

Exercise 3.a

(a) Write dealer's expected profits upon receiving buy/sell order, assuming informed trader follows his signal.

$$\mathbb{E}[\Pi_D | Buy] =$$

$$\mathbb{E}[\Pi_D | Sell] =$$

Exercise 3.a

(a) Write dealer's expected profits upon receiving buy/sell order, assuming informed trader follows his signal.

$$\mathbb{E}[\Pi_D|Buy] = \frac{\pi\frac{1}{2}\rho(a - v^H) + \pi\frac{1}{2}(1-\rho)(a - v^L) + (1-\pi)\frac{1}{2}(a-\mu)}{\pi\frac{1}{2}\rho + \pi\frac{1}{2}(1-\rho) + (1-\pi)\frac{1}{2}}$$
$$= \pi\rho(a - v^H) + \pi(1-\rho)(a - v^L) + (1-\pi)(a-\mu)$$

$$\mathbb{E}[\Pi_D|Sell] = \frac{\pi\frac{1}{2}\rho(v^L - b) + \pi\frac{1}{2}(1-\rho)(v^H - b) + (1-\pi)\frac{1}{2}(\mu - b)}{\pi\frac{1}{2}\rho + \pi\frac{1}{2}(1-\rho) + (1-\pi)\frac{1}{2}}$$
$$= \pi\rho(v^L - b) + \pi(1-\rho)(v^H - b) + (1-\pi)(\mu - b)$$

where $\mu = \frac{v^H + v^L}{2}$.

Exercise 3.b

(b) Compute the bid and ask prices set by a risk-neutral competitive dealer.

$$a =$$

$$b =$$

Exercise 3.b

(b) Compute the bid and ask prices set by a risk-neutral competitive dealer.

$$a = \pi\rho v^H + \pi(1-\rho)v^L + (1-\pi)\mu$$

$$b = \pi\rho v^L + \pi(1-\rho)v^H + (1-\pi)\mu$$

from $\mathbb{E}[\Pi_D|Buy] = \mathbb{E}[\Pi_D|Sell] = 0$.

Exercise 3.c

(c) Derive bid-ask spread as a function of signal's informativeness ρ. When is the market more or less liquid? Why?

$$S = a - b$$

Exercise 3.c

(c) Derive bid-ask spread as a function of signal's informativeness ρ. When is the market more or less liquid? Why?

$$\begin{aligned} S &= a - b \\ &= \pi(2\rho - 1)(v^H - v^L) \end{aligned}$$

Higher $\rho \Leftrightarrow$ larger $S \Leftrightarrow$ less liquid market because of larger adverse selection costs (same as larger π).

Exercise 3.d

(d) Verify that the speculator's strategy (buy after signal H, sell after signal L) is optimal.

Consider signal H:

$$\mathbb{E}[\Pi_I | H, Buy] =$$

$$\mathbb{E}[\Pi_I | H, Sell] =$$

Exercise 3.d

(d) Verify that the speculator's strategy (buy after signal H, sell after signal L) is optimal.

Consider signal H:

$$\begin{aligned}\mathbb{E}[\Pi_I|H, Buy] &= \rho(v^H - a) + (1-\rho)(v^L - a)\\ &= \left(\rho v^H + (1-\rho)v^L - \mu\right)(1-\pi) > 0\end{aligned}$$

$$\begin{aligned}\mathbb{E}[\Pi_I|H, Sell] &= \rho(b - v^H) + (1-\rho)(b - v^L)\\ &= (1-\pi)\left[\mu - (\rho v^H + (1-\rho)v^L)\right] - \pi(2\rho - 1)(v^H - v^L) < 0\end{aligned}$$

Same for signal L.

GM with Uniform value

- **Uniform outcome**: Suppose v is uniformly distributed on [0,1]
- **Prior value**: Prior density $f(v) = 1$, and $\mu = \mathbb{E}[v] = \int_0^1 vf(v)dv = 1/2$
- Look for an ask price $a < 1$:
  - Speculator buys if $v > a$ and sells if $v < b$ ($v = a$ has zero prob.). Thus:

$$\mathbb{P}(Buy|v) = \begin{cases} (1-\pi)\beta_B + \pi \text{ if } v > a; \\ (1-\pi)\beta_B \text{ if } v < a. \end{cases}$$

  - Recall Bayes' Rule: $f(v|Buy) = \frac{f(v)\mathbb{P}(Buy|v)}{\mathbb{P}(Buy)}$. Then,

$$f(v|Buy) = \begin{cases} \dfrac{(1-\pi)\beta_B + \pi}{(1-\pi)\beta_B + (1-a)\pi} & \text{if } v > a; \\ \dfrac{(1-\pi)\beta_B}{(1-\pi)\beta_B + (1-a)\pi} & \text{if } v < a. \end{cases}$$

GM with Uniform value (2)

Now we explicity have a on both sides of $a = \mathbb{E}[v|Buy]$. Must solve:

$$\begin{aligned}
a &= \mathbb{E}[v|Buy] \\
&= \int_0^a \frac{(1-\pi)\beta_B \cdot v}{(1-\pi)\beta_B + (1-a)\pi}\,dv + \int_a^1 \frac{[(1-\pi)\beta_B + \pi] \cdot v}{(1-\pi)\beta_B + (1-a)\pi}\,dv \\
&= \frac{(1-\pi)\beta_B \cdot a^2/2}{(1-\pi)\beta_B + (1-a)\pi} + \frac{[(1-\pi)\beta_B + \pi] \cdot (1-a^2)/2}{(1-\pi)\beta_B + (1-a)\pi} \\
&= \frac{(1-\pi)\beta_B + \pi - \pi a^2}{2(1-\pi)\beta_B + 2(1-a)\pi} = \frac{1}{2} + \frac{\pi a(1-a)}{2(1-\pi)\beta_B + 2(1-a)\pi}
\end{aligned}$$

This is a quadratic equation in a. For $\pi = \beta_B = 1/2$, solution is $a = \frac{3}{2} \pm \frac{\sqrt{3}}{2}$. Since we assumed $a < 1$, equilibrium in this case is $a = \frac{3}{2} - \frac{\sqrt{3}}{2}$.