Written Exam Economics summer 2022

Financial Markets Microstructure

June 9, 2022

This exam question consists of 04 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Final exam

Write up your responses to questions below and submit them to Digital Exam. The deadline to submit the responses is Jun 9, 22:00. No cooperation with other students is permitted.

Be concise, but show your work and explain your answers. Some questions may require you to make additional assumptions beyond those provided in the question; be clear about the assumptions you make. Some questions are open ended in that they may not have a unique correct answer. You are allowed to refer to textbooks, lecture notes, slides, problem sets, etc.

Problem 1: Twitter buyout

In April 2022, when Elon Musk has announced his intent to buy Twitter at \$54.20 per share, but no official acceptance from Twitter's Board of Directors has been issued, the following tweet has been made by the Saudi Prince Alwaleed bin Talal, head of Kingdom Holding Company, which owns 5.2 percent of Twitter:

> "I don't believe that the proposed offer by @elonmusk (\$54.20) comes close to the intrinsic value of @Twitter given its growth prospects ... I reject this offer."
> (`https://twitter.com/Alwaleed_Talal/status/1514615956986757127`)

Twitter stock closing price on NYSE on April 13th (day before the tweet) was \$45.85, and on April 14th (day of the tweet) was \$45.08.

1. Explain how the price movement on April 14th could be rationalized by the quoted tweet.
2. The Prince claimed the intrinsic value of Twitter was much higher than \$54.20 per stock, but the market price was significantly lower than that. Based on the course material, provide at least three possible explanations for this discrepancy.

Problem 2: Liquidity premium with liquidity shocks

When we looked at the Amihud-Mendelson model of illiquidity premium in class, we assumed that all traders have the same holding period h. This problem asks you to reconsider this model when instead of a fixed holding period h, all traders have a fixed probability λ with which a need to sell the asset arises in any given period.

In particular, consider an asset, whose (mid-)price grows at a constant rate R: $\mu_t = \mu_{t-1}(1+R)$, and which is traded with a constant relative half-spread $\frac{s}{2}$. Consider a market consisting of identical investors who consider buying the asset in period t at the current ask price $a_t = \mu_t\left(1+\frac{s}{2}\right)$. In every period $\tau > t$, if the investor is still holding the asset, a liquidity shock arrives with probability λ, which forces the investor to sell the asset at then-current bid price b_τ. The asset yields no dividends. The investor's outside option is putting money in a bank which yields return r and is a perfectly liquid investment.

1. Derive the nominal return R on the asset that should establish in equilibrium. (It should make the investors exactly indifferent between investing in the asset and pursuing their outside options.)
2. How does R depend on λ? On s? Explain. *NOTE: a formal argument coupled with an intuitive explanation is preferred. You can provide a purely intuitive argument for partial credit if you were unable to derive the exact expression for R.*

Problem 3: Dynamic limit order book with adverse selection: Effects of algorithmic trading

This problem explores the effects of informed trading in a version of the Parlour model that we have seen in class. Suppose that there is one asset, whose fundamental value v is unknown, and whose market valuation evolves according to $\mu_t = \mathbb{E}[v \mid \Omega_t] = \mu_{t-1} + \epsilon_t$, where $\epsilon_t \in \{-\sigma, \sigma\}$ with equal probabilities is period-t news, publicly announced at the end of period t (after any period-t orders are submitted).[1] In every period t, one risk-neutral trader arrives at the market. With probability π the trader is *informed* and already knows this period's news ϵ_t. With probability $1-\pi$ the trader is *uninformed* but has an idiosyncratic valuation $y_t \in \{-\sigma, \sigma\}$ with equal probabilities, which is independent of all $\{\epsilon_t\}$. The period-t uninformed trader thus values the asset at $v + y_t$.

Suppose that in every period, there is one ask price $a_t = \mu_{t-1} + S$ and one bid price $b_t = \mu_{t-1} - S$, where S denotes the half-spread, constant across periods. Each arriving trader can choose between submitting a limit order for one unit at the respective price or a market order against an existing order in the limit order book. A limit order is valid for one period and is automatically cancelled if it is not traded against by the next trader. Suppose further that a limit order submitted in period t can be automatically cancelled or repriced when ϵ_t is revealed, so a limit sell order submitted in period t is effectively priced at $a_{t+1} = \mu_t + S$, and a limit buy order at $b_{t+1} = \mu_t - S$. Let $d_t \in \{\varnothing, MS, LS, LB, MB\}$ denote the order submitted by period-t trader, where $d_t = \varnothing$ means the trader abstains from trading, and the other four denote, respectively, the market sell, limit sell, limit buy, and market buy orders.

1. What is the expected utility of a period-t *informed* trader from using a limit buy order, as a function of its execution probability p_{MS}?
2. What is the expected utility of a period-t *uninformed* trader from using a limit buy order, as a function of its execution probability p_{MS}?
3. What are the expected utilities that the informed and uninformed traders get from using a market buy order (assuming a limit sell order is in the book)?
4. Conjecture that when $y_t = +\sigma$, the *uninformed* trader uses a market buy order with probability α, assuming one is available, and a limit buy order w.p. $1-\alpha$; when $\epsilon_t = +\sigma$, the *informed* trader uses MB w.p. β and LB w.p. $1-\beta$; and symmetric strategies are used when $y_t/\epsilon_t = -\sigma$. Calculate the spread level S^U (as a function of α, β) that renders the uninformed traders indifferent between market and limit orders.
5. Calculate the spread level S^I (as a function of α, β) that renders the uninformed traders indifferent between market and limit orders. How does it compare to S^U? Explain this relation intuitively: which group of traders is more willing to provide liquidity and why?
6. Taking S as exogenous, what kind of pure-strategy equilibria can arise for different levels of S? (Note that relation of S and S^I, S^U determines α and β, which, in turn, determine S^I and S^U.)
7. Where do you expect the equilibrium spread S to be, relative to the interval that you identified. (Who has the power to determine S? Would these traders prefer higher or lower S? Calculate the equilibrium S if you can.)

   NOTE: you can attempt to answer this question intuitively for partial credit if you have not answered some or all of the parts 1-6 above.
8. Turns out, this problem was about algorithmic trading all along! In particular, suppose that it is exactly algorithmic trading that gives the limit traders their ability to reprice their limit orders before

[1] Object Ω_t denotes all public information available to the market at (the end of) period t.

they are picked off. Explain intuitively the implications of algorithmic trading in the context of this model.

NOTE: you are expected to make an educated guess about the results in the absence of algotrading; you are not expected to analyze the whole model without repricing. You can attempt to answer this question even if you have not answered some or all of the parts 1-7 above.