Final re-exam solutions

Problem 1: Inventory risk and demand for liquidity

Consider a Kyle model with inventory risk and no informed trading that we considered in class. How does the dealer's pricing schedule in that model depend on the variance of the incoming market order, σ_u^2? Explain the intuition behind this result and which modelling assumptions are responsible for this conclusion.

Solution

The dealer's pricing schedule in the mentioned model does not depend on σ_u^2 (see slide deck L7, slide 23). This is because the dealer's pricing decisions are guided by the premium they require for holding on to the inventory given the uncertainty in the asset valuation. There is no informed trading that attempts to hide between noise traders' orders. This model further assumes that the dealer maximizes their one-period-ahead wealth, which abstracts away from how the question of how the dealer will unravel their inventory. Finally, the dealers are assumed to be competitive. Therefore, there is no reason for the variance of liquidity demand to affect the pricing offered by the dealer. If, however, any of the aforementioned assumptions were altered, the result would be different.

Problem 2: Kyle model with public information acquisition

Consider a single-period Kyle model, where the speculator does not know the asset's fundamental value v perfectly, but instead decides how much to *publicly* invest in a noisy signal about v. In particular, suppose that before submitting an order, the speculator chooses σ_s^2, pays cost $c(\sigma_s^2)$, and then receives signal $s \sim \mathcal{N}(v, \sigma_s^2)$. All other agents in the market (specifically, the market-maker) observe the speculator's choice of σ_s^2.

After that, the game proceeds as in the regular Kyle model. The speculator chooses their trade size, $x \in \mathbb{R}$, to maximize their expected profit $\Pi_I \equiv \mathbb{E}[x(v-p)]$. The noise traders submit a random market order $u \sim \mathcal{N}(0, \sigma_u^2)$. The competitive dealer observes the aggregate order imbalance $q = x + u$ and quotes a price $p(q)$ at which they are willing to absorb it. All agents have a common prior belief that $v \sim \mathcal{N}(\mu, \sigma_v^2)$.

One can verify that for a given signal precision σ_s^2 and speculator's strategy $x(s) = \beta(s-\mu)$ for some β, the competitive dealer's price schedule is given by $p(q) = \mu + \lambda q$ with $\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\beta^2\sigma_s^2+\sigma_u^2}$. One can then verify that the speculator indeed optimally trades according to $x(s) = \beta(s-\mu)$ with $\beta = \frac{1}{2\lambda} \cdot \frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}$.

1. Give a plausible justification to the assumption that the speculator's choice of σ_s^2 is *observable* to other market participants.
2. Solve for equibrium speculator's trading aggressiveness β and the price impact λ in terms of model parameters and σ_s^2.
3. Calculate the speculator's expected trading profit for given σ_s^2, σ_v^2.
4. Suppose now the speculator's information cost is given by $c(\sigma_s^2) = \frac{\gamma}{\sqrt{\sigma_s^2}}$ for some information cost parameter γ. Derive the amount of information $\tau_s \equiv \frac{1}{\sigma_s^2}$ the speculator acquires as a function of $\lambda, \gamma, \sigma_v^2$.
5. How does the speculator's information choice depend on γ, σ_v^2, and σ_u^2 in equilibrium? Explain.
6. Answer intuitively: after committing publicly to some level of σ_s^2, would the speculator want to secretly change σ_s^2? Why or why not? Explain.

Solution

1. We can think of the speculator as a hedge fund, and of their information acquisition efforts as the size of their research department, in terms of headcount and funding. The funds would likely advertise information like this to attract clients, so it would not be a stretch to assume that other market participants can readily observe it.

2. We have

$$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \beta^2\sigma_s^2 + \sigma_u^2} \qquad \beta = \frac{1}{2\lambda}\cdot\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}$$

$$\Rightarrow \lambda = \frac{\sigma_v^2}{2\sqrt{\sigma_u^2(\sigma_v^2+\sigma_s^2)}} \qquad \beta = \sqrt{\frac{\sigma_u^2}{\sigma_v^2+\sigma_s^2}}$$

3. The speculator's expected trading profit is

$$\begin{aligned}
\mathbb{E}[x(v-p)] &= \mathbb{E}_s\left[\beta(s-\mu)\cdot(\mathbb{E}[v-\mu|s]-\lambda\beta(s-\mu))\right] \\
&= \mathbb{E}_s\left[\beta(s-\mu)\cdot\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}(s-\mu) - \frac{\sigma_v^2}{2(\sigma_v^2+\sigma_s^2)}(s-\mu)\right)\right] \\
&= \beta\frac{\sigma_v^2}{2(\sigma_v^2+\sigma_s^2)}(\sigma_v^2+\sigma_s^2) = \frac{\beta\sigma_v^2}{2} \\
&= \frac{\sigma_v^2}{2}\cdot\sqrt{\frac{\sigma_u^2}{\sigma_v^2+\sigma_s^2}},
\end{aligned}$$

since $\mathbb{E}[(s-\mu)^2] = \sigma_v^2+\sigma_s^2$.

4. The speculator's expected profit is given by trading profit net of information cost:

$$\frac{\sigma_v^2}{2}\cdot\sqrt{\frac{\sigma_u^2}{\sigma_v^2+\sigma_s^2}} - \frac{\gamma}{\sqrt{\sigma_s^2}}.$$

Maximizing that over σ_s^2, we get the First-Order Condition

$$\begin{aligned}
-\frac{\sigma_v^2\sigma_u}{4(\sigma_v^2+\sigma_s^2)^{\frac{3}{2}}} + \frac{\gamma}{2(\sigma_s^2)^{\frac{3}{2}}} &= 0 \\
\Rightarrow \tau_s = \frac{1}{\sigma_s^2} = \left(\frac{\sigma_u^2}{4\gamma^2\sigma_v^2}\right)^{\frac{1}{3}} &- \frac{1}{\sigma_v^2}.
\end{aligned}$$

5. We can see that the speculator acquires more information (chooses higher precision τ_s/lower variance σ_s^2) when:
   - γ is lower – information is cheaper;
   - σ_u^2 is higher – having more noise trades makes the market deeper (lower λ);
   - σ_v^2 is "average" – information is more valuable when the fundamental is more uncertain, but this is offset by the higher price impact that a more informed speculator faces. Therefore, as σ_v^2 increases, in equilibrium it sometimes pays off for the speculator to commit to a lower σ_s^2 in order to increase the market depth and mitigate the price impact.

6. Yes. On top of direct costs $c(\tau_s)$, the speculator in this model incurs indirect costs of τ_s, which stem from λ. Specifically, τ_s is factored by the market-maker into the depth of the pricing schedule the

speculator faces when trading – i.e., costs stem from the observability of τ_s or, in other words, from the market maker's expectation of the speculator's τ_s. In turn, the benefits of τ_s come from the actually chosen amount of information (not the other agents' expectation of it) – more private information about v means larger scope for profitable trades. Therefore, secretly increasing τ_s brings additional benefit but does not increase the "indirect costs".

Problem 3: Frozen Concentrated Orange Juice

Read a brochure about the Frozen Concentrated Orange Juice (FCOJ) future market attached at the end of this exam text. Answer the following questions.

1. According to the brochure, what are the two main goals of the FCOJ future market?
2. According to the brochure, what two types of traders participate in the FCOJ future markets? Which of these traders, do you think, are more likely to have informational advantage?
3. The figure on p.1 of the report shows that orange crop utilization for sake of producing FCOJ has been steadily declining in both absolute and relative terms during 1992–2011. The last Figure on p.3 of the report, however, shows that both trading volume and open interest in FCOJ futures has remained steady during that period. So the FCOJ market has been declining, but the FCOJ futures market has not. Propose an explanation for this discrepancy. How does it relate to market goals you identified in question 1?

Solution

1. Price discovery and risk transfer.
2. Commercial/hedging traders (related to citrus, juice-packing, or retail business) and speculative traders (everyone else). Speculators only have access to public information, while commercial traders have relevant insider information from their business activities. (It is, however, possible, that speculators are better at processing publicly available information than commercial traders, so they would have a different kind of informational advantage.)
3. While the relevance of FCOJ as a commodity has declined, this financial asset "remains the most visible price discovery mechanism for the [citrus] industry". I.e., this asset is relevant to other forms of juices and other types of citrus, and serves as a proxy asset for hedgers seeking to insure against risks in citrus/citrus juice markets more broadly, and enables price discovery in those markets.