Lecture 9

- solve exercise 3 in chapter 4 (p.159) which explores competition among speculators

Exercise 4.3

- Consider a Kyle model with many speculators $i \in \{1, 2, ..., N\}$.
- Every speculator uses a linear strategy

$$x_i = \beta(v - \mu)$$

- Everything else is same as in lecture

Exercise 4.3.a

(a) Find the equilibrium aggressiveness β, determine how it depends on N and explain why.

Speculator i maximizes the expected profit $\mathbb{E}[x_i(v-p)]$.

The clearing price is given by the zero-profit condition:

$$p = \mu + \lambda[x_i + (N-1)\beta(v-\mu) + u]$$

Plugging it into the expected profit expression and maximizing it w.r.t. x_i yields

$$x_i = \underbrace{\frac{1-\lambda\beta(N-1)}{2\lambda}}_{=\beta}(v-\mu)$$

$$\Rightarrow \beta = \frac{1}{\lambda(N+1)}$$

Exercise 4.3.a

$$\beta = \frac{1}{\lambda(N+1)}$$

- β decreasing in N: the more traders, the smaller is one trader's share (as in Cournot oligopoly)
- $N\beta$ increasing in N: total order size increases in N because with high N, each trader suffers less from higher price (due to lower output)

Exercise 4.3.b

(b) Derive the price impact coefficient λ from the dealer's zero-profit condition

$$\lambda = \frac{\mathbb{C}(v, q)}{\mathbb{V}(q)} = \frac{N\beta\sigma_v^2}{(N\beta)^2\sigma_v^2 + \sigma_u^2}$$

Combining with the expression for β, we get

$$\lambda = \frac{\sqrt{N}}{N+1}\frac{\sigma_v}{\sigma_u} \qquad \beta = \frac{1}{\sqrt{N}}\frac{\sigma_u}{\sigma_v}$$

Exercise 4.3.c

(c) What is the market depth in equilibrium, and how is it affected by an increase in the number of informed traders, N? What is the economic intuition for this result? Do you think this result is robust?

- Depth is $\frac{1}{\lambda} = \frac{N+1}{\sqrt{N}} \frac{\sigma_u^2}{\sigma_v^2}$
- It is increasing in N because traders become more aggressive (relative to their market share) in the presence of competition – submit larger orders $|x_i|$ given any fixed v, – price reacts less to any fixed order size.
- The conclusion is not 100% robust...

Exercise 4.3.d

(d) Compute the ex-ante expected profit of each informed investor. What is the effect of an increase in N on the aggregate profit of informed investors?

$$\mathbb{E}[x_i(v-p)] = \frac{\sigma_u \sigma_v}{(N+1)\sqrt{N}}$$

- predictably, the profits of each speculator decrease in N
- aggregate profit of all speculators $\frac{\sigma_u \sigma_v \sqrt{N}}{N+1}$ also decreases in N (because total aggressiveness $N\beta$ increases away from the monopolistic optimum)