

Financial Markets Microstructure

Lecture 2

Prices and Liquidity

Chapter 2 of FPR

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What did we do last week?

- 1 Introduced **financial markets** broadly speaking and motivate why we wanted to talk about it
- 2 Characterized **market formats**: order-driven markets (auctions) and dealer markets
- 3 Introduced some of the key **concepts and language**: dealer sets bid/ask price, traders submit market/limit orders

This lecture:

1 Fundamental value and Prices

2 Liquidity

3 Measures of Liquidity

4 Estimating Direction of Trade

5 Estimating Quotes

6 Other Measures of Liquidity

7 Implementation Measures

Fundamental Value

- We'd like to believe there is some “objective” / “fundamental” value of a stock – at least to some representative agent.
- The question of whether to buy or sell then often amounts to:
“Is the current asset price above or below its fundamental value?”

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“Is the current asset price above or below its fundamental value?”
- To answer, need to understand what **fundamental value** is.
- **Short answer:** **expected discounted cash flow**. Affected by many factors:
 - 1 R+D
 - 2 Governance
 - 3 Marketing
 - 4 Competition
 - 5 ...

Also affected by investors' preferences (discounting, risk-aversion, income profile)

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- Long answer: study [asset pricing](#).
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- Question of **market microstructure**: take the 'fundamental value' as given and analyze how it **translates into prices** in realistic markets.
- Dual question: **price discovery**
"How much information about the fundamental value can be extracted from market prices?"

Fundamental Value 2

- Long answer: study **asset pricing**.
 - Field of finance devoted to calculating asset price relative to other assets, assuming perfect markets.
- Question of **market microstructure**: take the ‘**fundamental value**’ as given and analyze how it **translates into prices** in realistic markets.
 - In GameStop case, divergence was due to bubble(?)
 - Another broad reason for divergence: **limited liquidity**
- Dual question: **price discovery**
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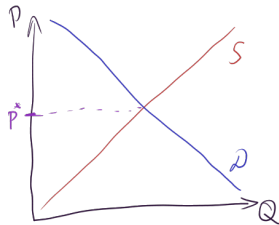
6 Other Measures of Liquidity

7 Implementation Measures

What is liquidity?

■ In perfect markets:

- There is one price = valuation cutoff
- Agents who value asset above the cutoff end up with it (keep or buy)
- Agents who value asset less end up without it (sell or do not buy)
- This is the **efficient** allocation that we want



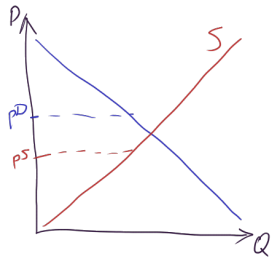
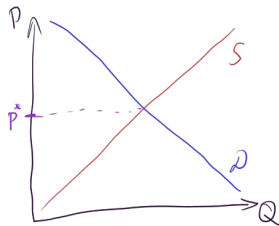
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■ In (financial and many other) real markets:

- Bid/ask prices different from that ideal cutoff/fundamental value
- (due to limited liquidity)
- Allocation **inefficient**



Still, what exactly is liquidity?

- **Market liquidity** = “market’s ability to facilitate an asset being sold quickly (for cash) without having to reduce its price very much (or even at all)”
 - Not everyone who wants to trade in a given asset is present in the market at the same time
 - Liquidity depends not just on exogenous parameters, but also traders’ endogenous behavior

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 - Liquidity depends not just on exogenous parameters, but also traders’ endogenous behavior
- Do not confuse with (related notions of):
 - 1 **Funding liquidity** = “economic agent’s ability to obtain cash/credit at acceptable terms, to meet obligations without incurring large losses”
 - Banks are ‘liquidity constrained’ when they do not have enough cash on hands to meet demand for withdrawals (despite having enough assets)
 - You are liquidity constrained when your wage arrives in two days but you need to pay your rent today.
 - 2 **Monetary liquidity** = “asset’s ability to be exchanged for goods”
 - Assets in the order of decreasing liquidity: cash, checking deposits, long-term deposits, housing, ...

Why do we care about liquidity?

- Traders: liquidity provides a measure of trading costs, affects how costly it is to implement a given theoretical trading strategy
- Regulators:
 - 1 Efficiency is tricky to measure in financial markets: liquidity provides a proxy
 - 2 Illiquid markets also seem to be more prone to medium-run price deviations from fundamentals
 - 3 Illiquidity *may* be a sign of structural problems in the market
- Relation to **depth**: depth measures how much must be traded to move price by certain amount
 - \approx sensitivity of liquidity to trade size

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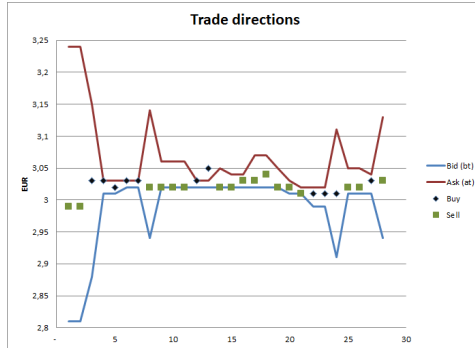
Liquidity measures

Liquidity is not a very well-defined concept, so it's not immediate how to measure it either. We will consider several measures:

- 1 **Spread measures:** quoted spread, effective spread, realized spread
- 2 **Implementation measures:** volume-weighted average price, price impact, implementation shortfall
- 3 **Non-trading measures:** trading volumes
- 4 *Missing data estimators:* Lee-Ready algorithm for trade direction, Roll's measure for quotes

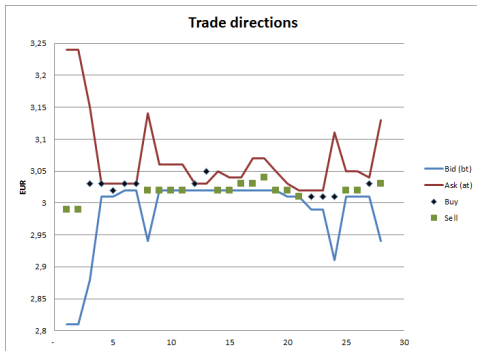
Example

- We will play around with a dataset on KrispyKreme stock



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- Notice that price is sometimes inside spread: price improvements (either hidden limit orders or price improvement given by dealer)
- Also a price outside spread (recall bid/ask only valid for x units)

Quoted spread

- Let a_t (b_t) be the best ask (bid) price at time t

- Quoted spread:

$$S_t = a_t - b_t$$

- Contemporaneous: spread facing trader at time t

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- Normalize to get normalized quoted spread

$$s_t = \frac{S_t}{m_t},$$

where m_t is the midprice:

$$m_t = \frac{a_t + b_t}{2}.$$

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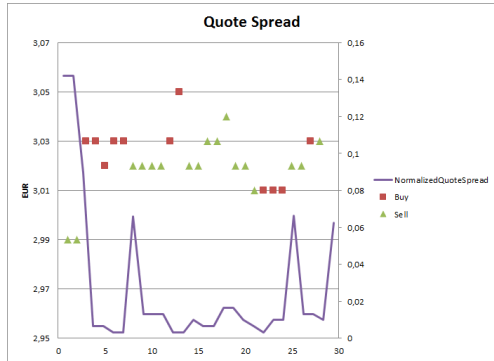
- We can generalize it to consider average spread for trade size q :

$$S_t(q) = \bar{a}_t(q) - \bar{b}_t(q)$$

where $\bar{a}_t(q)$ and $\bar{b}_t(q)$ are average execution prices

Quoted spread

- Applying the definition to the data, we get:



- Notice that the quoted spread does not capture price improvements (for instance in the first three observations)

Effective spread

- Suppose one market order is executed per period, and
 - d_t : trade direction (1: buyer-initiated, -1: seller initiated)
 - p_t : price

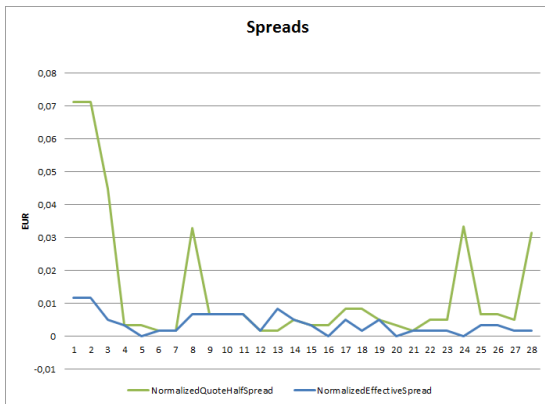
- Effective (half-)spread:

$$S_t^e = d_t(p_t - m_t),$$
$$s_t^e = \frac{S_t^e}{m_t}$$

- Backward looking: spread faced by previous trader
- Compare actual price with midquote the instant before: measures price impact and captures 'price improvements'

Effective spread

- Apply to data and compare to quoted spread



- Effective spread is often lower (since it captures price improvements); can be larger if there was a big market order

Realized spread

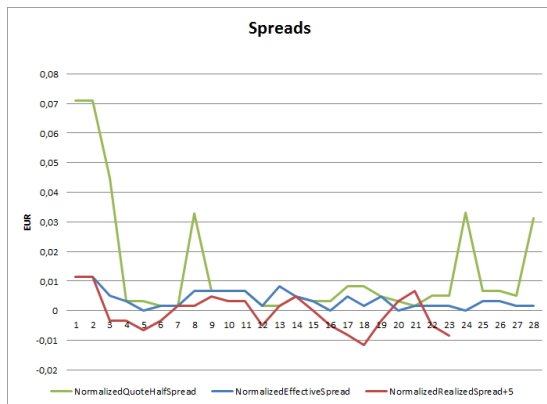
- Realized spread:

$$\begin{aligned} S_t^r &= d_t(p_t - m_{t+\Delta}) \\ &= d_t(p_t - m_t) - d_t(m_{t+\Delta} - m_t) \end{aligned}$$

- Spread realized by somebody who holds the asset for Δ periods
- Idea: measure the spread after prices have adjusted to new information
- As a forward-looking measure:
 - $\mathbb{E}_t S_t^r = d_t(p_t - m_t) = S_t^e$ if $\mathbb{E}_t m_{t+\Delta} = m_t$
- As a backward-looking measure:
 - Typically smaller than effective spread: why?

Realized spread

- Calculate for $\Delta = 5$ and compare to other measures



- Realized spread is indeed often lower than effective spread

Comparing the spreads

- The **quoted spread** and the **effective spread** may be more useful to traders:
 - Quoted spread: what is the quoted trading cost now
 - Effective spread: what was the trading cost faced by the last trader

These are (imperfect) measures of the cost of executing a market order now

- The **realized spread** is more relevant to a market maker (liquidity provider) or a researcher:
 - It measures the cost of taking a position (long or short) for an amount of time
 - Can also be interpreted as the long-run (informational) impact of trades on prices

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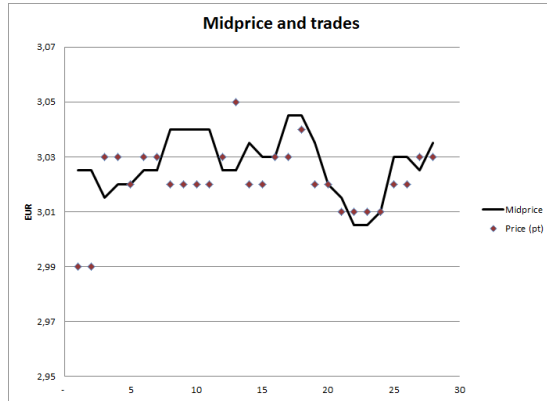
Estimating direction of trade

- We often only observe quotes and realized prices: not the direction of trade
- Thus, we need to develop methods to classify trades
- Complication: trading may be 'within the quotes': harder to guess direction
- **Lee-Ready algorithm:** (Lee and Ready [1991])

$$d_t = \begin{cases} 1 & \text{if } |p_t - a_t| < |p_t - b_t| \\ & \text{or } p_t = m_t \text{ and } p_t > p_{t-1} \\ -1 & \text{if } |p_t - a_t| > |p_t - b_t| \\ & \text{or } p_t = m_t \text{ and } p_t < p_{t-1} \end{cases}$$

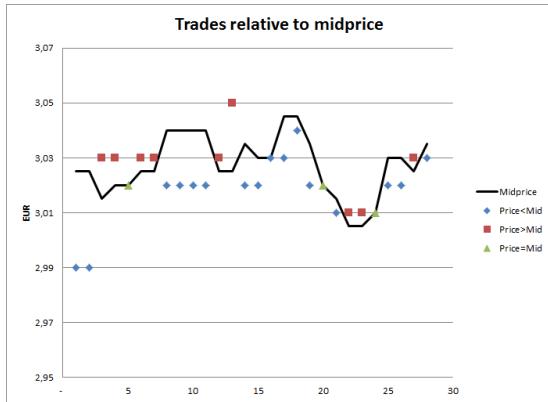
Estimating Lee-Ready

First, we calculate midprices and compare to trade prices



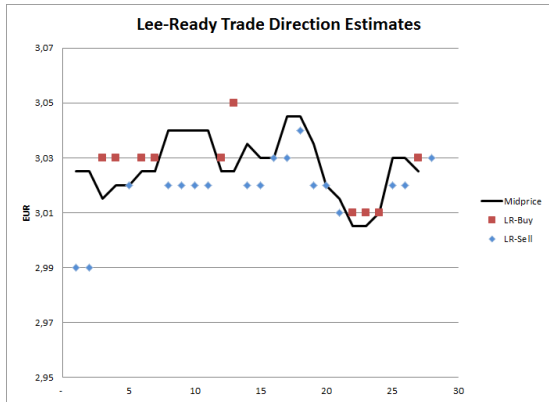
Estimating Lee-Ready (2)

Then we compare trade prices to midprices



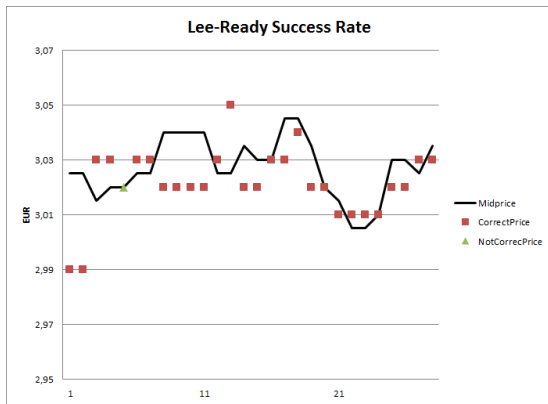
Estimating Lee-Ready (3)

Finally, we classify the trades that were just on the midprice



Estimating Lee-Ready (4)

Checking with the actual trade directions, we see that we only made one mistake



Lee-Ready precision

- Odders-White [2000]: large-scale (144 NYSE stocks over 3 months; $> 400k$ transactions) test of Lee-Ready algorithm with NYSE data
- 85% correct
- Most mistakes with:

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- 85% correct
- Most mistakes with:
 - trades at the midpoint
 - small transactions
 - transactions in large-cap / frequently traded stocks

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- We often lack information on quotes to compute the spread
 - Can we estimate the spread knowing only trade prices?
- Roll [1984]: use transaction prices to estimate it
 - 1 Construct a simple model of trading and calculate spread
 - 2 Estimate it
 - 3 Check robustness to simplifying assumptions

Roll's model

Suppose the following:

- 1 All trades have the same size. $d = 1$: buy, $d = -1$: sell
- 2 Arriving orders are i.i.d. with $\mathbb{P}(d_t = 1) = \frac{1}{2}$
- 3 Midquote is random walk: $m_t = m_{t-1} + \epsilon_t$, where ϵ_t are i.i.d. shocks
- 4 Market orders are not informative: $\mathbb{E}(d_t \epsilon_t) = \mathbb{E}(d_t \epsilon_{t+1}) = 0$
- 5 Spread $S = a_t - b_t$ is constant.

Then

$$p_t = m_t + \frac{d_t S}{2}.$$

We know p_t but not m_t . How do we estimate S ?

Roll's model

- Roll's observed that although ϵ_t and d_t are i.i.d., $\Delta d_t = d_t - d_{t-1}$ is mean-reverting, yielding:

$$\text{Cov}(\Delta p_t, \Delta p_{t-1}) = \text{Cov}(p_t - p_{t-1}, p_{t-1} - p_{t-2})$$

- Intuitively: if $\Delta d_t > 0$, this means that we go from a sale to a buy - then the next change must be opposite

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The estimator

- We can then work out that

$$\text{Cov}(\Delta p_t, \Delta p_{t-1}) = -\frac{S^2}{4},$$

giving us the estimator

$$S_t^R = 2\sqrt{-\text{Cov}(\Delta p_t, \Delta p_{t-1})}.$$

- Recall the assumptions of the model. We (the book) can work out extensions to treat some of them
- In our example: $S_t^R = 0.01$

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Price impact

- How much do trades affect prices? **Price impact** λ ; $1/\lambda$ captures market *depth*

$$\Delta m_t = \lambda q_t + \epsilon_t.$$

Here q_t is the **order imbalance** in period t . In our example: $\lambda = 0.15$ (q_t in 100,000EUR)

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- **Hasbrouck measure** (γ): sensitivity of returns to trading volume (Hasbrouck [2007])

$$|\Delta m_t| = \gamma Vol_t + \epsilon_t.$$

Meaningful for single trades, but if t aggregates many trades, γ is hard to interpret.

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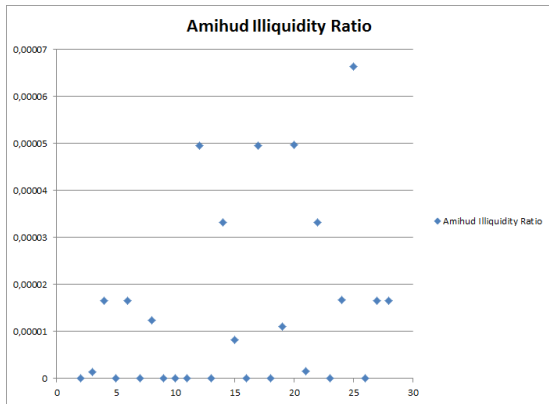
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- **Amihud measure** (I_t): take ratio btw return Δm_t and volume to get *illiquidity ratio*: (Amihud [2002])

$$I_t = \frac{|\Delta m_t|}{Vol_t}.$$

Amihud's Illiquidity Ratio



Somewhat volatile on high-frequency data. Usually calculated for daily data and/or then averaged over a longer interval (month) – but then it is also hard to interpret.

On Hasbrouck and Amihud measures

- Neither Hasbrouck, nor Amihud measures make immediate sense when applied to aggregate data – yet this is the most common application.
- Afaik, at least the Hasbrouck measure was born to deal with pre-1983 historical stock data, which only contained aggregated daily prices and volumes, and no intraday data.
- Hasbrouck [2007] shows that γ is mildly correlated with λ under some distributional assumptions
- Bottom line: do not use γ or I if you have data that lets you estimate λ directly.

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Implementation Measures

We can also look at measures of how costly it is for traders to implement a given trade in reality, as opposed to “paper trading” (looking at trades-you-could-have-done).

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Example

[Perold (1988) observed that] from 1965 to 1986, a paper portfolio based on the Value Line ranking system outperformed the market by 20% per year, and the real Value Line fund, which implemented the trades recommended in the newsletter, outperformed the market by only 2.5% per year, emphasizing that the quality of implementation is at least as important as the investment idea itself. [Anand et al., 2012]

Volume based measure

- Volume-Weighted Average Price (VWAP):

$$VWAP = \sum w_i p_i,$$

where $w_i = |q_i| / \sum_i |q_i|$ is the order weight, q_i is the size of order i

- This equals the amount of dollars traded over the number of shares traded: average price
- Trader can compare the price he got with VWAP to evaluate how good was his deal relative to market.
- This measure may depend excessively on few orders (if they are large) and therefore be subject to manipulation
- For our example, $VWAP = 3.02$

Implementation shortfall

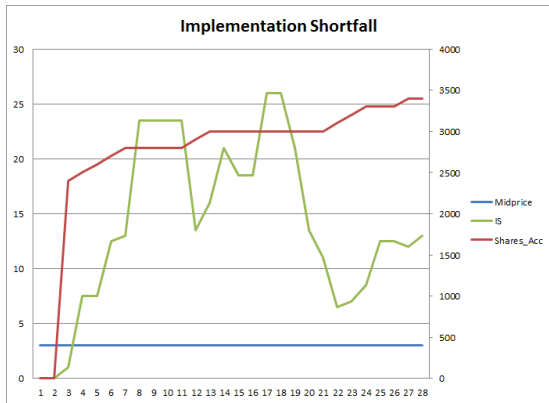
- Aim at time 0: to (net) purchase q shares at current (mid)price m_0
 - Suppose by time t , fraction κ_t has been executed, at an average execution price \bar{p}_t
 - The realized trading gain is $\kappa_t q(m_t - \bar{p}_t)$
 - An ideal gain from immediate execution without price impact would have been $q(m_t - m_0)$
 - The difference is the **implementation shortfall**:

$$\begin{aligned} IS_t &= q(m_t - m_0) - \kappa_t q(m_t - \bar{p}_t) \\ &= \kappa_t q(\bar{p}_t - m_0) + (1 - \kappa_t)q(m_t - m_0). \end{aligned}$$

- Interpretation: Execution cost plus opportunity cost

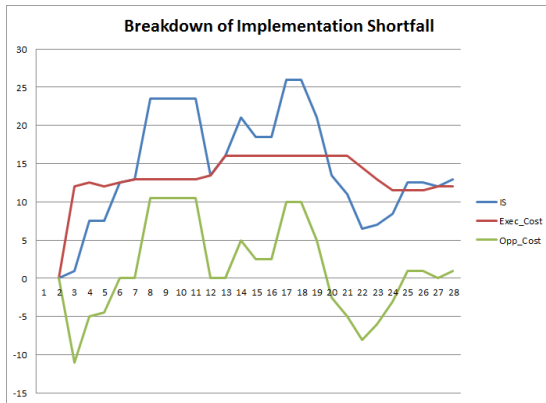
Implementation shortfall

- Suppose in the KrispyKreme example you want to buy 3,500 shares
- And suppose all the buy orders in the data (3,400 shares) came from you



Implementation shortfall

Breaking down the shortfall into opportunity cost and execution cost



Implementation shortfall

- Anand et al. [2012] show evidence that top market performers have a consistently negative implementation shortfall.
- “there is more to a trading strategy than just selecting a broker”
 - (e.g., when to trade, how much, how to react to market movements...)

Other measures

- Measures such as **trading volume** and **trade frequency** are also used
- **Time-to-trade** for limit orders is another measure, but difficult to use (depends on order size, depends on past traders' intent – some post LOs to provide liq-ty, trading is not the final goal)
- Some measures may contradict each other, e.g.:
 - trading volume and spreads are both positively correlated with information releases (why?)
 - price volatility is low in very liquid – but also very illiquid markets
- Frequency of trading or related measures may be more relevant in 'thin' markets, for instance in emerging economies

Conclusion

- We have looked at different manners in which to estimate liquidity
- No method is perfect: depends on trade size, time horizon, trade motivation
- Data shows that liquidity varies both continuously throughout a trading day, and more abruptly around big events
- Next time we will start analyzing *what* drives the spread

Exercises for next week

- Recreate the graphs and figures and numbers I presented today using the KrispyKreme dataset. Better: calculate the (average) values of all measures for the whole dataset.
- Solve exercise 8 regarding implementation shortfall, on page 75 in the textbook. Discuss the meaning of m_t in this analysis.

References I

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