

Financial Markets Microstructure

Lecture 6

Liquidity and Price Dynamics
Chapter 3.4-3.7 of FPR

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What did we do last week?

- 1 Information and prices
- 2 Efficiency and markets
- 3 Glosten and Milgrom: Workhorse model to analyze adverse selection in markets
 - Analysis of what drives the spread
 - Tradeoff between market liquidity and price discovery
 - The model had reasonably good efficiency properties

Today

- 1 Look at other drivers of the spread
 - Order-processing costs
 - Dealer inventory risk
- 2 We'll look at how their dynamic effect on prices differ

3

What order-processing costs exist?

A liquidity supplier (for instance a dealer) can have a range of different order-processing costs

- Trading fees: charged by exchanges
- Clearing and settlement fees: paid if a central clearinghouse is used to minimize trading risks
- Overheads: back office expenses
- (Dealer rents)

These costs must somehow be compensated by traders, and will therefore enter the spread

5

How do these costs affect the spread?

- Let $\mu_t \equiv \mathbb{E}[v|\Omega_t]$ be the expectation of v *after* the time- t trade order is observed, and let s_t^a and s_t^b denote the 'half-spreads'
- Hence, μ_{t-1} represents what we now when period t starts
- Then in the GM model we can write prices as

$$a_t = \mu_{t-1} + s_t^a$$

$$b_t = \mu_{t-1} - s_t^b$$

- Assume dealer has order cost γ , and charges this directly to trader:

$$a_t = \mu_{t-1} + \gamma + s_t^a$$

$$b_t = \mu_{t-1} - \gamma - s_t^b$$

6

How do these costs affect the spread? (2)

- Hence, the new bid-ask spread is

$$S_t = a_t - b_t = 2\gamma + s_t^a + s_t^b$$

- The spread is now made up of order costs (2γ) and adverse selection costs ($s_t^a + s_t^b$)
- Suppose we want to determine whether spread in a given market is due to adverse selection or order costs
 - The instantaneous effect of order costs is similar to that of adverse selection costs
 - But we shall see that the dynamic effect is different

7

The dynamics of the spread

- As before, let $d_t = 1$ denote a buyer-initiated trade, and $d_t = -1$ a seller-initiated trade
- Also, let $s(d_t)$ be the adverse-selection-related half-spread depending on the trade:
 $s(1) = s_t^a$ and $s(-1) = s_t^b$
- Then the realized price can be written as

$$p_t = \mu_{t-1} + (s(d_t) + \gamma)d_t$$

- Since $\mu_t = \mu_{t-1} + s(d_t)d_t$, then

$$p_t = \underbrace{\mu_t}_{\text{updated valuation}} + \underbrace{\gamma d_t}_{\text{order cost}}$$

8

The dynamics of the spread

Then the effect of time- t trade on prices:

- **short-run:**

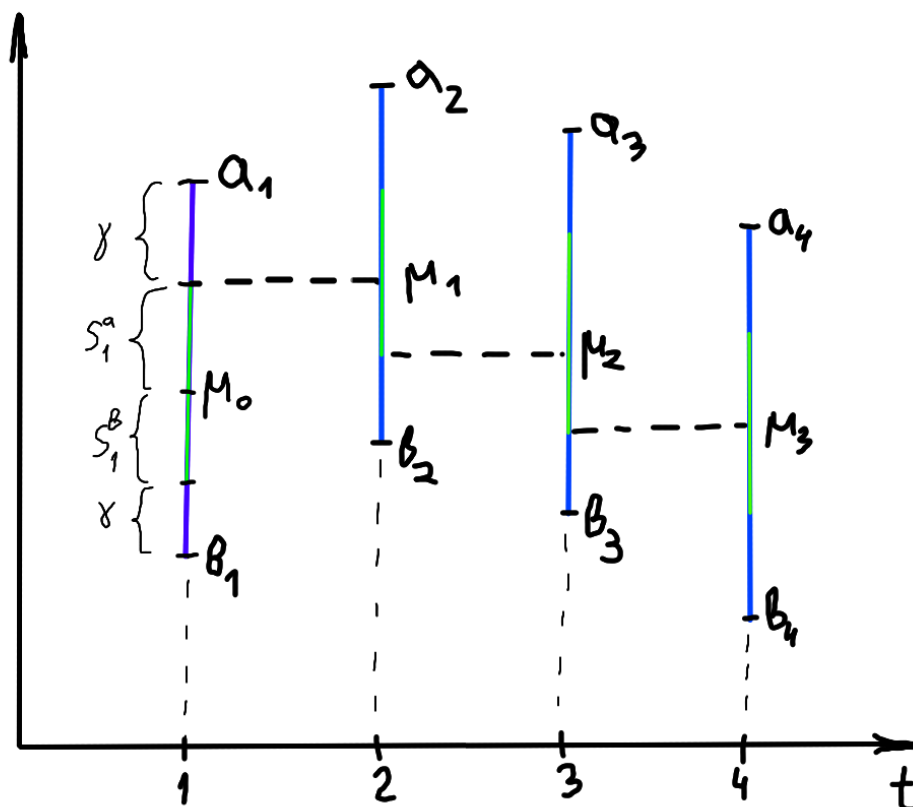
$$p_t - \mu_{t-1} = (s(d_t) + \gamma)d_t$$

- **long-run:**

$$\begin{aligned}\mathbb{E}_t[p_{t+s}] - \mu_{t-1} &= \mathbb{E}_t[\mu_{t+s-1} + (s(d_{t+s}) + \gamma)d_{t+s}] - \mu_{t-1} \\ &\approx \mathbb{E}_t[\mu_{t+s-1}] - \mu_{t-1} \\ &= \mu_t - \mu_{t-1} \\ &= s(d_t)d_t\end{aligned}$$

- so order cost effect on prices is *transient* and is reversed by future trades;
- effect of adverse selection term is *permanent*

9



10

Inventory risk

- Illiquidity can arise due to dealers' asset **inventory cost**
 - Holding inventory is **risky**, so dealers adjust their quotes to unwind any accumulated inventory.
- Textbook illustrates this using Stoll (1978) model
 - But this model is from pre-game theory times and has a strange solution method
 - So we will instead **extend the Glosten-Milgrom model** (extension not in the book)

12

Glosten-Milgrom model: Inventory risk edition

- Suppose for simplicity there are **no speculators** ($\pi = 0$)...
- ...but there are public news about asset value: $\mu_{t+1} = \mu_t + \epsilon_t$ (where $\mu_t = \mathbb{E}[v|\Omega_t]$)
- Noise traders behave as usual
- **Dealer** has some **inventory** z_t of the stock and c_t of cash
- Dealer is **risk averse**
 - For concreteness, assume mean-variance preferences over next-period wealth:

$$U(w_{t+1}) = \mathbb{E}[w_{t+1}] - \frac{\rho}{2} \mathbb{V}(w_{t+1}),$$

where $w_t = z_t \mu_t + c_t$ and $\rho > 0$ measures risk aversion

- (equivalent to CARA expected utility preferences when returns are normal)

13

GM-IRE: Dealer's utility

- As usual, let $\mathbb{E}[\epsilon_t] = 0$, $\mathbb{V}(\epsilon_t) = \sigma^2$, and remember that z_t is the current inventory, and $w_t = z_t \mu_t + c_t$.
- Dealer's utility at the end of period t is

if no orders: $U(w_{t+1}|N) =$

if Buy order: $U(w_{t+1}|B) =$

if Sell order: $U(w_{t+1}|S) =$

14

GM-IRE: Quotes

To derive equilibrium **quotes**, use the **zero-profit condition**

- i.e., ensure that the dealer's expected utility does not change after trading
- (assume all competing dealers have same inventory z_t)

$$U(w_{t+1}|B) = U(w_{t+1}|N) \Rightarrow a_t =$$

$$U(w_{t+1}|S) = U(w_{t+1}|N) \Rightarrow b_t =$$

15

GM-IRE: Quotes (2)

- **Spread** is $S_t = \rho\sigma^2$
 - Positive due to dealer's risk-aversion
 - Increasing in risk-aversion coeff ρ and asset value volatility σ^2
- **Midquote** is $m_t = \mu_t - \rho\sigma^2 z_t$
 - Depends on dealer's inventory z_t . Dealer demands risk premium for taking a position in the asset.
 - Effects depend on risk paratements ρ and σ^2
 - To emphasize: prices here are **not efficient**
 - This inefficiency would in principle also motivate traders to submit the "right" orders – arbitrage!
(Not present in this model, since we assumed traders are not strategic and not sensitive to the price.)

16

Summary

- The spread is driven not only by adverse selection: order costs and inventory risk have an effect as well
 - Price efficiency is lost once these new factors come into play
- How can we figure out which of these factors are more/less important? By their long-term effects! Will talk about that soon.
- But before we go there, next time: what affects market *depth*?

17

Homework

We said today that inventory risk is priced when the dealer is risk-averse. Risk-aversion is one explanation, but other factors can also contribute to inventory risk. The two following cases explore this issue:

- A big trader was punted off the Nordic power market after failing to meet margin calls (two articles on [absalon](#)).
 - How does inventory risk manifest in this story?
 - Explain why such inventory risk can be priced even by risk-neutral agents.
- Negative oil futures prices were registered in 2020 (blog post on [absalon](#) or [here](#)).
 - Why did it happen? How do negative prices make sense?
 - How does inventory risk manifest in this story?
 - Explain why such inventory risk can be priced even by risk-neutral agents.

18