Ch 8, ex 2

Price discovery and transparency. Consider the model of post-trade transparency described in section 8.2. Consider the time-averaged expected squared deviation between the transaction price and the true value of the security, that is

$$\frac{E\left[(p_1^k - v)^2\right]}{2} + \frac{E\left[(p_2^k - v)^2\right]}{2},$$

where p_t^k is the transaction price in period $t = 1, 2$ in regime $k = T, O$ (transparent, opaque). Show that price discovery is more efficient in the transparent market. You may limit your analysis to the case $\pi > \frac{1}{2}$ in which the equilibrium first-period spread is positive.

Post-trade transparency

- If orders arrive sequentially, what effect does information about past orders have?
- **Value**: high v^H or low v^L with equal probability
  - Mean: $\mu = (v^H + v^L)/2$
  - Denote $\sigma = (v^H - v^L)/2$
- **Dealers**: set quotes, competitive, risk neutral
- **Traders**: two traders arrive, submit unit market orders
  - With prob. π: both are informed
  - With prob. $(1-\pi)/2$: both liquidity traders; first seller, then buyer
  - With prob. $(1-\pi)/2$: both liquidity traders; first buyer, then seller
- **Transparent market**: All dealers observe the first order y_1
  - Set $a_1 = \mu + \pi(v^H - \mu)$ and $a_{2y_1} = \mathbb{E}[v|y_1, buy]$

Post-trade transparency: Period 2

Opaque market: First dealer gains informational advantage. Focus on ask side

- Period 2. Denote the dealer who observed period-1 trade by I, and the other dealer by U.
  - For technical reasons, suppose I sets price after observing U's quote
  - Dealer U: How to quote if you didn't see the first trade and second trade is buy?
    - If you set ask $a_2^U < v^H$ you will be undercut by I if first order was a sell
    - You only get to trade if first order was buy: lose $v^H - a_2^U$
    - Thus, uninformed dealers need to quote $a_2^U = v^H$
  - Dealer I: Suppose you saw the first trade, and second trade is a buy:
    - Set price at $a_{2s}^I = v^H$ if first trade was a sell, and $a_{2b}^I = v^H - \epsilon$ if buy
    - I wins period-2 buy order if y_1 was a sell (otherwise can undercut U)
    - U wins period-2 buy order if y_1 was a buy, since I knows that asset value is high

Post-trade transparency: Period 1

- Period 1. The sequential information advantage uncovered in the previous slide can make dealers bid keenly for the first order (Forex dealers often said to quote negative spread to large traders)
  - In second period, I's profit is $(1-\pi)(v^H - v^L)/2$. U's profit is zero
  - Competition leads the first period half-spread to be reduced by this amount, to $(2\pi - 1)(v^H - v^L)/2$ (dealers undercut each other to obtain information contained in first order)
  - The uninformed's aggregate trading cost is $\pi(v^H - v^L)$ - double the cost under transparency. Why is this?
- Would dealers commit to transparency?
  - No, there is always an incentive to hide your orders (section 8.4.2)
  - May explain the rise of less transparent trading venues

Ch 8, ex 2

- Transparency, $t = 1$:

$$a_1^T = \mu + \pi\sigma \qquad\qquad b_1^T = \mu - \pi\sigma$$

$$\begin{aligned}
E\left[(p_1^T - v)^2\right] =& \frac{1}{2}\left[\pi(a_1^T - v^H)^2 + (1-\pi)\frac{1}{2}(a_1^T - v^H)^2 + (1-\pi)\frac{1}{2}(b_1^T - v^H)^2\right] + \\
&+ \frac{1}{2}\left[\pi(b_1^T - v^L)^2 + (1-\pi)\frac{1}{2}(b_1^T - v^L)^2 + (1-\pi)\frac{1}{2}(a_1^T - v^L)^2\right] \\
=& (1-\pi^2)\sigma^2
\end{aligned}$$

Ch 8, ex 2

- $t = 2$:
  - $p_2 = v$ if informed at $t = 1$,
  - $p_2 = \mu$ if uninformed at $t = 1$;

$$\Rightarrow E\left[(p_2^T - v)^2\right] = \pi \cdot 0 + (1-\pi)\sigma^2$$

- Averaging over time:

$$\left[\frac{1}{2}(1-\pi^2) + \frac{1}{2}(1-\pi)\right]\sigma^2 = (1-\pi)\left(1+\frac{\pi}{2}\right)\sigma^2$$

Ch 8, ex 2

- Opaqueness, $t = 1$ (assuming $\pi > 1/2$ to avoid crossed quotes):

$$a_1^O = \mu + (2\pi - 1)\sigma \qquad\qquad b_1^O = \mu - (2\pi - 1)\sigma$$

$$E\left[(p_1^O - v)^2\right] = 2(1 - \pi)\sigma^2$$

Ch 8, ex 2

- Opaqueness, $t = 1$ (assuming $\pi > 1/2$ to avoid crossed quotes):

$$a_1^O = \mu + (2\pi - 1)\sigma \qquad b_1^O = \mu - (2\pi - 1)\sigma$$

$$E\left[(p_1^O - v)^2\right] = 2(1 - \pi)\sigma^2$$

- $t = 2$:

$$E\left[(p_2^O - v)^2\right] = 2(1 - \pi)\sigma^2$$

- so the average is also $2(1 - \pi)\sigma^2$

Ch 8, ex 2

Comparison:

$$\frac{E\left[(p_1^T - v)^2\right]}{2} + \frac{E\left[(p_2^T - v)^2\right]}{2} < \frac{E\left[(p_1^O - v)^2\right]}{2} + \frac{E\left[(p_2^O - v)^2\right]}{2}$$

$$(1-\pi)\left(1+\frac{\pi}{2}\right)\sigma^2 < 2(1-\pi)\sigma^2$$

$$1+\frac{\pi}{2} < 2$$

Transparency yields better price discovery