

## Written Exam Economics summer 2023

### Financial Markets Microstructure

June 8<sup>th</sup> 2023

This exam question consists of 12 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

***The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.***

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Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
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- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
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## Final re-exam

Write up your responses to questions below and submit them to Digital Exam. The deadline to submit the responses is Aug 28, 21:00. No cooperation with other students is permitted.

Be concise, but show your work and explain your answers. Some questions may require you to make additional assumptions beyond those provided in the question; be clear about the assumptions you make. Some questions are open ended in that they may not have a unique correct answer. You are allowed to refer to textbooks, lecture notes, slides, problem sets, etc.

### Problem 1: Spread dynamics in Glosten-Milgrom model

Consider a standard Glosten-Milgrom model:

- asset fundamental value is  $v \in \{v^L, v^H\}$ , the two realizations are considered equally probable ex ante;
- a competitive dealer sequentially quotes bid  $b_t$  and ask  $a_t$  prices for one unit of the asset each period;
- one trader arrives at the market per period and can submit a market buy or sell order for one unit or do nothing:  $d_t \in \{-1, 0, 1\}$ ;
- with probability  $\pi$  the trader is informed and knows  $v$  and chooses  $d_t$  to maximize profit; with complementary probability  $1 - \pi$  the trader is uninformed and submits either a buy or a sell order with equal probabilities regardless of  $v$ .

Answer the following questions.

1. Calculate the first-period market valuation  $\mu_0 = \mathbb{E}[v]$ , the ask and bid prices  $a_1, b_1$ , and the relative spread  $s_1 = \frac{a_1 - b_1}{\mu_0}$ .
2. Suppose the first order was a sell:  $d_1 = -1$ . Calculate the second-period market valuation  $\mu_1 = \mathbb{E}[v | d_1 = -1]$ , the ask and bid prices  $a_2, b_2$ , and the relative spread  $s_2 = \frac{a_2 - b_2}{\mu_1}$ .
3. How does  $s_2$  compare to  $s_1$ ? Give an intuitive explanation for why. Do you expect this trend to continue from  $s_2$  to  $s_3$  and onwards?

### Problem 2: Dynamic LOB markets with naive traders

This problem explores a version of the Foucault/Parlour model that we have seen in class. Suppose that there is one asset, whose fundamental value  $v$  is unknown, and whose market valuation evolves according to  $\mu_t = \mathbb{E}[v | \Omega_t] = \mu_{t-1} + \epsilon_t$ , where  $\epsilon_t \in \{-\sigma, 0, \sigma\}$  with equal probabilities is period- $t$  news, publicly announced at the end of period  $t$  (after any period- $t$  orders are submitted).<sup>1</sup> In every period  $t$ , one risk-neutral trader arrives at the market (who only knows  $\mu_{t-1}$  but not  $\epsilon_t$ , and has no idiosyncratic preference for the asset).

Suppose that in every period, there is one ask price  $a_t = \mu_{t-1} + S$  and one bid price  $b_t = \mu_{t-1} - S$ , where  $S$  denotes the half-spread, constant across periods. Each arriving trader can choose between submitting a limit order for one unit at the respective price or a market order against an existing order in the limit order book. A limit order is valid for one period and is automatically cancelled if it is not traded against by the next trader.<sup>2</sup> Let  $d_t \in \{\emptyset, MS, LS, LB, MB\}$  denote the order submitted by period- $t$  trader, where  $d_t = \emptyset$  means the trader abstains from trading, and the other four denote, respectively, the market sell, limit sell, limit buy, and market buy orders.

Assume first as usual that all traders are strategic and profit-maximizing.

<sup>1</sup>Object  $\Omega_t$  denotes all public information available to the market at (the end of) period  $t$ .

<sup>2</sup>To be clear: a limit order submitted in period  $t$  can **not** be cancelled or repriced when  $\epsilon_t$  is revealed.

1. What is the expected utility of a period- $t$  trader from using a limit buy order, as a function of its execution probability  $p_{MS}$ ? What about a market buy order?
2. Derive the period- $t$  trader's optimal trading strategy as a function of  $\epsilon_{t-1}$  and  $S$ .  
*Hint: it might be useful to consider cases  $S = 0$ ,  $S \in (0, \sigma)$ ,  $S = \sigma$ , and  $S > \sigma$ .*
3. Explain why in equilibrium with trade it should be that  $S = \sigma$ . Explain intuitively how the equilibrium looks, why this should be the market-clearing price, and what the traders' equilibrium profits are.

Now, assume instead that all traders are *naive* in that they do not account for adverse selection when submitting limit orders. That is, when they submit a limit order, they expect that the asset's value conditional on trade is  $\mu_{t-1}$  (on average).<sup>3</sup>

4. What is the subjective expected utility of a period- $t$  naive trader from using a limit buy order, as a function of its execution probability  $p_{MS}$ ? What about a market buy order?
5. Derive the period- $t$  naive trader's equilibrium trading strategy and the respective trading probabilities in an equilibrium with  $S > 0$ .  
*Bonus: characterize the set of equilibria as fully as you can.*
6. Compare the equilibrium you found in part 5 to the equilibrium from part 3. Explain intuitively how they are different and what drives the difference between the two.

### Problem 3: He liked the bonds

Read the article on AMI bonds attached at the end of this exam.<sup>4</sup>

You are to take the role of a financial market regulator in an internal discussion about this case (e.g., a SEC analyst making a presentation to your colleagues). Write a memo discussing this case, with an emphasis on the following:

1. How did the described manipulation affect market participants?
2. What kinds of remedies can you suggest to mitigate such exploits in the future?
3. What kinds of side effects could your remedies have?

*NOTE: you can use the help of chatbots/AI/LLMs such as chatGPT. If you do, state clearly how they were used and which parts of the answer are mainly written by a LLM and which by you.*

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<sup>3</sup>The traders still estimate trading probabilities  $p_{MS}$ ,  $p_{MB}$  correctly.

<sup>4</sup>This text is a part of a Bloomberg opinion piece, available at <https://www.bloomberg.com/opinion/articles/2023-04-10/ftx-lost-track-of-its-money>. You should ignore the text on the first page before the headline "He liked the bonds" and all text on the last page after the headline "APE Endgame".

|with no further context.

Incredibly relatable. Who among us has not saved our most precious data in a file titled “use this one”? This works, I guess, imperfectly, if your company is basically a half-dozen friends in a suite in the Bahamas. But it’s not best practices, and in particular, if the friends who run the company all get kicked out for incompetence and criminality and angry professionals come in to clean up the mess, they will be very annoyed to find all the crypto stored like this.

## He liked the bonds

One fun little financial story of 2017 and 2018 involved the bonds of American Media Inc., the publisher of the National Enquirer. AMI was majority-owned by a hedge fund firm named Chatham Asset Management LLC. It had some bonds outstanding. Those bonds did not trade much, but they traded some, and they kept trading at higher and higher prices. By November 2017, some AMI bonds were trading at lower yields than the bonds of Apple Inc. [2](#) I am sure AMI is a fine tabloid publisher but it is not Apple, and its bond prices were perplexing. Also in 2018 AMI tried to raise some more money [by selling more bonds](#), and it eventually sold them at a 10.5% interest rate, which was much higher than the trading yield of its existing bonds. To the naked eye, AMI looked like a small, high-yield newspaper publisher, and when it sold new bonds the prices reflected that. But its *old* bonds traded in the secondary market at prices that made it look more creditworthy than Apple. What gives?

Everyone kind of knew the answer, which is that the AMI bonds didn’t trade that much, and they seemed to trade mostly *within*

*Chatham:*

Chatham, the majority equity owner of AMI's stock, also owned most of its bonds, and sometimes Chatham would sell the bonds to itself at ever-increasing prices. This didn't cost Chatham much — it was basically just overpaying itself — and made AMI's credit look good. But of course when AMI wanted to raise money by selling more bonds, Chatham was not going to buy those bonds at a 2% interest rate; it wanted a regular interest rate, like 10.5%.

This is not particularly legal, and last week Chatham and its founder Anthony Melchiorre [settled with the US Securities and Exchange Commission](#), agreeing to [pay \\$19 million](#) for doing this. *Why* did Chatham do this? The story that [the SEC tells](#) is basically that Chatham liked the bonds a lot and didn't want to let them go. Chatham ran a bunch of different accounts — some hedge funds, some liquid alternatives funds — and held large AMI bond positions in all of them. Sometimes some accounts would need to sell AMI bonds: They had concentration limits that meant they couldn't have too much of their money in AMI bonds, or customers took money out and Chatham needed to sell bonds to pay them. When this happened, Chatham did not want to let the bonds go, so it sold them to other Chatham accounts so they could stay in the family. [The SEC says](#):

Generally, when Chatham was forced to sell a high conviction AMI Bond in these circumstances, Chatham desired to purchase the AMI Bond for another Client because Chatham still believed in the merits of the investment and would not otherwise be selling the AMI Bonds if it were not for the portfolio restrictions or cash needs of the selling Clients.

As a general matter, selling bonds to yourself is allowed, but awkward: If you really believe in some bonds but have to sell

them from one account, and you want to buy them in another account, you are not *forbidden* from doing so, but there are lots of ways for it to look bad. Generally the ways for it to look bad are (1) you overpay for the bonds, making money for the selling account at the expense of the buying account (and causing the reported trading price of the bonds to be wrong) or (2) you underpay for the bonds, making money for the buying account at the expense of the selling account (and causing the reported trading price of the bonds to be wrong). The solution is generally to sell yourself the bonds at a fair market price, neither underpaying nor overpaying. Broadly speaking two ways to do that are:

1. Sell the bonds into the market at market prices, and then buy them back a bit later from the market at market prices: You don't trade with yourself at all, but only with arm's-length counterparties; or
2. Figure out a fair market price using outside sources (trading pries, pricing services, quotes from dealers, etc.) and sell the bonds to yourself at that price.

The first option is probably better, but it requires an active market; if you're the only real buyer or seller of the bonds it's hard.

Anyway Chatham sort of ... waved in the direction of doing this the right way?

Recognizing that there were legal restrictions on trading between RICs [3](#) and their affiliates, which included other Chatham Clients, Chatham and Melchiorre sought advice from a compliance consultant on how to facilitate the Rebalancing Trades. The consultant advised Chatham to conduct the trading either through a single broker over more than one day or

through multiple brokers if on the same day. The foundational principle underlying the advice was to ensure that the transactions occurred at independently-derived market prices.

Yeah I mean that's good generic advice, but when you are the market for the bonds it doesn't work out great. Also Chatham did not necessarily go all in on the *spirit* of that advice:

Around the time that Chatham began to execute the Rebalancing Trades, Melchiorre generally explained the purpose of the Rebalancing Trades to the Rebalancing Brokers. Melchiorre informed each of the Rebalancing Brokers to whom he sold a Client's AMI Bonds that he likely would have an interest in repurchasing that same AMI Bond he was selling for another Client. Over time, an understanding developed on the part of the Rebalancing Brokers that whenever Melchiorre placed an order to sell one of the AMI Bonds for a Client, he would repurchase it for another Client, either directly the following day or days, or indirectly through another broker.

The Rebalancing Brokers engaged in the Rebalancing Trades because they expected Melchiorre to repurchase the bonds. The business model of several of the Rebalancing Brokers was to "match" buy and sell orders from their customers. Those Rebalancing Brokers ordinarily did not purchase securities for their own inventory—i.e., put the firm's own capital at risk—or they did so on a very limited basis. Nonetheless, the vast majority of the Rebalancing Trades involved at least one Rebalancing Broker that purchased bonds into its firm's inventory. For example, some of the Rebalancing Brokers would at times agree to purchase securities from Chatham even though the Rebalancing Broker may not have lined up the other leg of the transaction. These brokers' willingness to do so was based on their expectation that Chatham would repurchase the

bonds, either directly or through another broker.

The purchasing Rebalancing Brokers generally did not offer the AMI Bonds to other customers in the market. Instead, in virtually every case, they resold the securities to Chatham or to another broker who they understood was purchasing for Chatham.

As Chatham's need to conduct rebalancing in its various Client Accounts increased over time, Rebalancing Trades became routine. When Melchiorre wanted to sell an AMI Bond to one particular Rebalancing Broker ("Rebalancing Broker A") and then repurchase it the following day, he would send Rebalancing Broker A a message indicating that he wanted to sell an AMI Bond in the "usual drill." Rebalancing Broker A then would purchase the AMI Bond into the firm's inventory until Melchiorre repurchased it the following day.

In the abstract, selling bonds to a broker one day and then buying them back for a different account a day or two later could be a good way to do everything at arm's-length market prices: The broker will pay, and charge, prices that reflect market levels; it won't overpay to buy from you or undercharge to sell to you.

But in practice, if you are the only buyer and the only seller and you call up a broker and say "hey it's the usual drill," you are not really getting a fair market price. The broker doesn't care what the buying price or selling price is, as long as you pay a commission. You can just pick whatever price you want:

Melchiorre proposed the price for the Rebalancing Trades and the Rebalancing Brokers agreed to it without first soliciting bids from other market participants.

When proposing a price for Rebalancing Trades, Melchiorre considered a number of factors, which included the prior day's price as reflected in prices published by a pricing service. Those



published prices would have been influenced, to some extent, by Chatham's own trading. When purchasing the AMI Bonds, Melchiorre also added a spread to compensate the Rebalancing Brokers. For example, in the case of a Rebalancing Trade executed through a single Rebalancing Broker overnight, Chatham would repurchase the AMI Bonds for a small spread above what it had sold the AMI Bonds to the broker the day before.

One consequence of this is that the compliance consultant's basic idea of selling to brokers to get market prices was not really working. Another consequence is that the prices kept going *up*: The only trades were Chatham's trades, and it kept trading at higher prices to compensate its brokers. Eventually this became absurd:

Over the Relevant Period, Chatham and Melchiorre engaged in over one hundred Rebalancing Trades in AMI Bonds, accounting for approximately 81 percent, on average, of the customer trading (i.e., not broker-to-broker trading) in such securities.

Over time, the frequent Rebalancing Trades and repeated mark-ups to compensate the Rebalancing Brokers resulted in the market price of AMI Bonds increasing at a faster rate than prices of similar securities. For example, by November 2017, two of the AMI Bonds traded in Rebalancing Trades at implied yields lower than the prevailing London Interbank Offered Rate ("LIBOR"). Such yields ordinarily would have been associated with a bond of a much higher creditworthiness than the AMI Bonds.

A third consequence is that Chatham's assets under management kept going up: It charged clients fees based on the value of its assets, and as it kept increasing the prices of these

bonds, that value kept going up. The SEC says:

Chatham was compensated for its advisory services to the [hedge] Funds with a management fee and a performance fee, and to the [liquid alts funds] with a management fee. The management fee charged to certain Clients was set at a percentage of the [net asset value] of those Clients.

In order to calculate each Fund's NAV, Chatham used an independent pricing service to determine the value of each of the portfolio securities, including the AMI Bonds. The LAF administrators calculated their own NAVs, also using the same pricing service Chatham used.

Chatham and Melchiorre understood that the pricing service that Chatham and the LAF administrators utilized was based to some extent on recent trading prices and that the Rebalancing Trades accounted for virtually all of the trading in the Bonds during the Relevant Period. Because the Rebalancing Trades at times increased the prices of the AMI Bonds, the NAVs of the Client accounts also were increased by that amount on those occasions.

Accordingly, the Clients paid Chatham an estimated \$11,000,000 in performance and/or management fees that they would not have in the absence of Chatham's Rebalancing Trades. Chatham in turn paid approximately 55 percent of such fees to Melchiorre.

Part of the \$19 million settlement is paying back that \$11 million to investors. But the SEC doesn't quite say that this is *why* Chatham did this. You could argue that this sort of thing — trading bonds back and forth with yourself in a way that raises their prices — is market manipulation, that the goal was to raise prices to charge higher fees. But the SEC doesn't say that:

It says, more or less in so many words, that the reason for the trading is that Chatham really liked the bonds, that it “still believed in the merits of the investment” even when it had to sell them, so it kept buying them back. The higher prices and higher fees were just a happy byproduct.

## **APE Endgame**

You know the story. AMC Entertainment Holdings Inc. became a meme stock, so it sensibly sold a ton of stock to raise money and pay down debt. Eventually it [ran out of stock to sell](#): Its corporate charter authorizes about 524 million shares of common stock, and it has sold basically all of them.

Shareholders did not seem interested in amending the charter to authorize more shares, because they were worried about dilution and/or because they are retail investors who tend not to vote their shares at all. The way it works is that a majority of the *outstanding shares* need to approve the charter amendment to issue new shares, so not voting is the same as voting no.

But AMC’s charter also allows the board to issue “blank-check” preferred stock, that is, preferred stock with any terms the board wants. So AMC [started issuing a new type of preferred stock called APEs](#), AMC Preferred Equity Units, which are meant to be identical to the common stock: They have the same economic rights, same voting rights, etc. AMC did a quasi-stock-split in which shareholders got one APE for each common share they held, and then it started selling new APEs to raise more money.

Part of the plan here was just to sell APEs to raise money, but another [part of the plan](#) was to [get the APEs to vote to amend the charter](#) to allow AMC to issue more common shares. If that happens, the APEs will all be converted into common shares; since now the APEs trade at a discount to the common, this will