

# Financial Markets Microstructure

## Lecture 19

Markets and public information

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### Previously on FMM

- **High-Frequency Trading** generates informational asymmetries between traders
- If the markets are already reasonably good at matching traders with opportunities, fast trading may be strictly bad for welfare
- While HFTs can provide liquidity, more HFTs does not necessarily improve liquidity
- So it might be optimal to eliminate the speed game, e.g. by moving away from continuous markets to frequent batch auctions

## News

- Incomplete information has been pivotal in many of the models we have considered
- But we really focused on **asymmetric information**
  - When **private news** arrive to insiders, they **trade**, everyone else learns from trades
- What about **symmetric uncertainty**? When **public** news arrive: ...?
  - Glosten-Milgrom/Glosten/Foucault: all orders are repriced, but **no trade should** take place
  - in reality, a lot of trade after public announcements
  - (Budish-Cramton-Shim offer one explanation: HFTs rush to snipe stale quotes when news arrive)
- Let's look at how information is aggregated and how this depends on public info!

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## Context: Hellwig [1980]

- A model of how disagreement leads to trading. Forget about liquidity takers/makers.
- Suppose there is a continuum of ex ante symmetric traders  $i \in [0, 1]$  with CARA prefs:

$$U(W_i) = -e^{-\gamma W_i}, \quad W_i = (v - p)x_i$$

- One asset, fundamental value  $v \sim \mathcal{N}(0, 1/\tau_v)$
- Every trader gets a private signal  $\eta_i = v + \epsilon_i$  with  $\epsilon_i \sim \mathcal{N}(0, 1/\tau_\epsilon)$  and submits a price-contingent demand schedule  $x_i(p)$
- Aggregate supply  $u \sim \mathcal{N}(0, \sigma_u^2)$  (provided by noise traders)
- Price  $p$  is set to clear the market (by some non-trading market-maker/“the invisible hand”/...)  $\rightarrow p$  is informative about average signal

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## Hellwig: eqm

- Conjecture a linear eqm, where price is a linear function of  $v$  and  $u$  (and aggregate signal  $\bar{\eta} = \int \eta_i di$ , which is irrelevant given  $v$  and  $u$ )
- $v|p, \eta_i$  is normal with some mean and variance  $\Rightarrow$  so is  $W_i$  (without noise trades,  $v|p$  is degenerate)
- Then trader  $i$ 's problem is:

$$\begin{aligned} & \max_{x_i} \left\{ \mathbb{E} \left[ -e^{-\gamma W_i} | \eta_i, p \right] \right\} \\ (W_i \text{ normal}) & \iff \max_{x_i} \left\{ \mathbb{E} [W_i | \eta_i, p] - \frac{\gamma}{2} \mathbb{V} [W_i | \eta_i, p] \right\} \\ & \Rightarrow x_i(\eta_i, p) = \frac{\mathbb{E} [v | \eta_i, p] - p}{\gamma \mathbb{V} [v | \eta_i, p]}, \quad p = \mathbb{E} \left[ \mathbb{E} [v | \eta_i, p] \mid p \right] - \gamma \mathbb{V} [v | \eta_i, p] u \end{aligned}$$

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## Hellwig: conclusion

- $\mathbb{E} [v | \eta_i, p] = a\eta_i + bp$  for  $a, b > 0$
- Traders **respond to private signals**: high  $\eta_i \Rightarrow$  buy, low  $\eta_i \Rightarrow$  sell
- This is because  $i$  believes that when  $\eta_i > p$ , this might be because high supply  $u$  depressed price, hence a purchase is justified (traders **disagree about the fundamental** value  $v$ )
- What if the traders care not about  $v$ , but the resale value instead?

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## Brown and Jennings [1989]

- Now consider the same economy but in dynamics: there are two periods  $t = 1, 2$ , two populations of traders with CARA preferences:
  - “early traders”  $i \in [0, 1]$ ;
  - “late traders”  $j \in [0, 1]$ ;
  - early traders need to offload their asset holdings to late traders:

$$W_i = (p_2 - p_1)x_i,$$

$$W_j = (v - p_2)x_j$$

- Private signals  $\eta_i = v + \epsilon_i$ ,  $\eta_j = v + \epsilon_j$
- Asset aggregate supply  $u_1, u_2$  i.i.d. normal in the two periods

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## Brown-Jennings: eqm

- In  $t = 2$ , same as Hellwig:

$$x_j(\eta_j, p_2, p_1) = \frac{\mathbb{E}[v|\eta_j, p_2, p_1] - p_2}{\gamma \mathbb{V}[v|\eta_j, p_2, p_1]}, \quad p_2 = \mathbb{E}\left[\mathbb{E}[v|\eta_j, p_2, p_1] \mid p_2, p_1\right] - \gamma \mathbb{V}[v|\eta_j, p_2, p_1] u_2$$

- In  $t = 1$ , instead of  $(v|\eta_i, p_1)$ , traders now care about  $(p_2|\eta_i, p_1)$ :

$$x_i(\eta_i, p_1) = \frac{\mathbb{E}[p_2|\eta_i, p_1] - p_1}{\gamma \mathbb{V}[p_2|\eta_i, p_1]}$$

i.e., their demand depends on  $\mathbb{E}[\mathbb{E}[v|\eta_j, p_2, p_1]|\eta_i, p_1]$  – their expectation of later investors' expectation of  $v$ .

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## Brown-Jennings: conclusions

- **Second-order beliefs** are important for trading volumes in dynamic settings.
- In this specific model, early traders have no reason to believe that late traders' estimate differs from their own:

$$\mathbb{E}[\mathbb{E}[v|\eta_j, p_2, p_1]|\eta_i, p_1] = \mathbb{E}[v|\eta_i, p_1],$$

hence early traders only trade (amongst themselves) for the same reason as before – they **disagree about the resale value**.

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## Brown-Jennings: public signals

- Revealing a **public signal**  $\nu$  about  $v$  at  $t = 1$  would make traders **agree more** (they would put less weight on  $\eta_i$ ), hence there would be **less trade** at  $t = 1$ ; the less trade, the more informative  $\nu$  is.
- But in reality, there is a lot of trading when public news are revealed (Bailey, Karolyi, and Salva [2006]). **Potential explanations** include:
  - Announcements are made up of public **and private** signals (in the presence of HFTs, public news  $\approx$  HFTs' private news)
  - Or traders have **heterogeneous priors** and can therefore 'agree to disagree'; announcements then can amplify or mitigate these initial disagreements and so generate trade.
- Turns out, there's another explanation: if you craft a more elaborate information structure, you can generate disagreement from public news!

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## Kondor [2012]: Example

- Two groups of traders again,  $I$  and  $J$
- Fundamental value has two components:  $v = v_I + v_J$
- I-trader signal:  $\eta_i = v_I + \epsilon^i$
- J-trader signal:  $\eta_j = v_J + \epsilon^j$
- Public signal:  $\nu = v + \epsilon^p$
- Suppose  $v_I, v_J, \epsilon^i, \epsilon^j, \epsilon^p$  are independent and normal with zero mean

Main features: public signal about all of  $v$ , private signals about *different aspects* of the fundamentals,  $v_I$  and  $v_J$ .

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## Example: Before public announcement

### Benchmark: No public news ( $\nu$ not observed)

- Traders' beliefs about  $v$  are

$$\mathbb{E}(v|\eta_j) = \mathbb{E}(v_J|\eta_j) + \mathbb{E}(v_I|\eta_j) = a_J\eta_j + 0, a_J > 0$$

$$\mathbb{E}(v|\eta_i) = \mathbb{E}(v_J|\eta_i) + \mathbb{E}(v_I|\eta_i) = a_I\eta_i + 0, a_I > 0$$

- I-trader's second-order belief is

$$\mathbb{E}(\mathbb{E}(v|\eta_j)|\eta_i) = \mathbb{E}(a_J\eta_j|\eta_i) = 0,$$

i.e.,  $\mathbb{E}(\mathbb{E}(v|\eta_j)|\eta_i) = \mathbb{E}(v)$  because  $\eta_i$  and  $\eta_j$  are independent

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## Example: With public announcement

### Given public signal $\nu$

- Traders' beliefs about  $\nu$  are

$$\mathbb{E}(\nu|\eta_j, \nu) = \mathbb{E}(\nu_J|\eta_j, \nu) + \mathbb{E}(\nu_I|\eta_j, \nu) = (b_J\eta_j + c_J\nu) + x_J\nu,$$

$$\mathbb{E}(\nu|\eta_i, \nu) = \mathbb{E}(\nu_J|\eta_i, \nu) + \mathbb{E}(\nu_I|\eta_i, \nu) = (b_I\eta_i + c_I\nu) + x_I\nu,$$

where  $b_k, c_k, x_k > 0$  and  $b_k < a_k$ .

First-order beliefs of  $i$ -traders **converge** due to public signal

- But  $I$ -agent's second-order belief is

$$\begin{aligned}\mathbb{E}(\mathbb{E}(\nu|\eta_j, \nu)|\eta_i, \nu) &= b_J\mathbb{E}(\eta_j|\eta_i, \nu) + (c_J + x_J)\nu, \\ &= b_J(e\nu - f\eta_i) + (c_J + x_J)\nu\end{aligned}$$

where  $e, f > 0$ .

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## Example: Conclusion

- $\mathbb{E}(\mathbb{E}(\nu|\eta_j, \nu)|\eta_i, \nu)$  is **decreasing in  $\eta_i$**  and the weight on  $\eta_i$  *increases* with the precision of  $\nu$
- i.e., second-order beliefs **diverge** among  $i$ -traders: the more precise  $\nu$  is, the less  $I$ -traders agree about the resale value of the asset  $\Rightarrow$  more trade among  $i$ !
- This disagreement generates trade after public signals.
- The remainder of the slides presents the Kondor's trading model and derivations in *slightly* more detail.

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## Kondor [2012]: Full(er) Model

### ■ Timing:

- 1 I-traders observe their information and trade
- 2 I-traders liquidate all their positions and sell to J-traders
- 3  $v$  (distributed as before) is realized and J-traders consume asset

### ■ Traders: Price takers, $i, j \sim U(0, 1)$ , demand $x_i(p_t)$ , util $u(W_i) = -e^{-\gamma W_i}$ and wealth

$$W_I = (p_2 - p_1)x_I; \quad W_J = (v - p_2)x_J.$$

### ■ Supply: Time- $t$ asset supply $u_t$ (from noise traders):

$$u_1 \sim \mathcal{N}(0, 1/\delta_1^2), \quad u_2 \sim \mathcal{N}(0, 1/\delta_2^2) \quad (u_2 \equiv u_1 + \Delta u_2)$$

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## Analysis: Trader maximization

### ■ Random supply: Implies that prices are not perfectly informative.

### ■ I traders: Solve

$$\max_{x_I} \mathbb{E} \left[ -e^{-\gamma W_I} | \eta_i, \nu, p_1 \right]$$

### ■ CARA utility and normal distributions $\Rightarrow$ can rewrite I traders' problem as

$$\max_{x_I} \left\{ \mathbb{E} [W_I | \eta_i, \nu, p_1] - \frac{\gamma}{2} \mathbb{V} [W_I | \eta_i, \nu, p_1] \right\}$$

### ■ J traders: Solve

$$\max_{x_J} \left\{ \mathbb{E} [W_J | \eta_j, \nu, p_1, p_2] - \frac{\gamma}{2} \mathbb{V} [W_J | \eta_j, \nu, p_1, p_2] \right\}$$

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## Analysis: Trader maximization (2)

- Taking the FOC and solving for the demands we get

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma} (\mathbb{E}[p_2 | \eta_i, \nu, p_1] - p_1), \quad (1)$$

$$x_{2,j}^* = \frac{\tau_v^2}{\gamma} (\mathbb{E}[v | \eta_j, \nu, p_1, p_2] - p_2), \quad (2)$$

where  $\tau_{p_2}^2 = 1/\mathbb{V}(p_2 | \eta_i, \nu, p_1)$  and  $\tau_v^2 = 1/\mathbb{V}(v | \eta_j, \nu, p_1, p_2)$

- In order to calculate expectations, need to make a conjecture about prices

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## Analysis: Linear prices and price signals

- **Equilibrium:** Look for equilibrium with linear price function/demand

$$p_1 = \frac{1}{e_1} [a_1 v_I + c_1 \nu - u_1] \quad (3)$$

$$p_2 = \frac{1}{e_2} [b_2 v_J + c_2 \nu + \eta_2 \mu_1 - u_2], \quad (4)$$

for some  $a_1, b_2, c_1, c_2, e_1, e_2, \eta_2$ , where  $\mu_1$  is the **price signal** of  $v_I$

- **Price signal:** This tells us the information contained in prices:

$$\mu_1 \equiv \mathbb{E}[v_I | p_1, \nu] = \frac{e_1 p_1 - c_1 \nu}{a_1} = v_I - \frac{1}{a_1} u_1; \quad (5)$$

$$\mu_2 \equiv \mathbb{E}[v_J | p_1, p_2, \nu] = \frac{e_2 p_2 - c_2 \nu - \eta_2 \mu_1}{b_2} = v_J - \frac{1}{b_2} u_2. \quad (6)$$

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## Analysis: Reformulating in terms of price signals

- **Rewrite expectation.** All variables jointly normal  $\rightarrow$  linear expressions

$$\mathbb{E}[p_2|\eta_i, \nu, p_1] = a_1^e \eta_i + b_1^e \nu + c_1^e \mu_1; \quad (7)$$

$$\mathbb{E}[\nu|\eta_j, \nu, p_1, p_2] = a_2^e \eta_j + b_2^e \nu + c_2^e \mu_1 + d_2^e \mu_2. \quad (8)$$

- **Rewrite FOC.** Plugging (7) and (8) into (1) and (2) we get

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma} (a_1^e \eta_i + b_1^e \nu + c_1^e \mu_1 - p_1); \quad (9)$$

$$x_{2,j}^* = \frac{\tau_v^2}{\gamma} (a_2^e \eta_j + b_2^e \nu + c_2^e \mu_1 + d_2^e \mu_2 - p_2). \quad (10)$$

- **Market clearing:**  $u_1 = \int_0^1 x_{1,i}^* di$  and  $u_2 = \int_0^1 x_{2,j}^* dj$  determine  $p_1$  and  $p_2$  resp.

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## Analysis: Equilibrium

- **Matching coefficients:** From market clearing, can show that  $p_t$  is linear function as conjectured
- **Equilibrium demand:** Matching up all the coefficients, we can then show that

$$x_{1,i}^* = a_1 \eta_i + c_1 \nu - e_1 p_1; \quad (11)$$

$$x_{2,j}^* = b_2 \eta_j + c_2 \nu + \eta_2 \mu_1 - e_2 p_2. \quad (12)$$

- Demand is increasing in private signal ( $\eta_i/\eta_j$ ), in public signal ( $\nu$ ), in price signal ( $\mu_1$ ), and decreasing in price ( $p_t$ ) (recall that traders are price takers)

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## Results: Demand period 2

- Let's look at what drives the agents' demands. Rewrite period-2 demand as

$$x_{2,j}^* = b_2(\eta_j - \mu_2) \quad (13)$$

- Notice that  $\eta_j$  is  $j$ 's private signal and  $\mu_2$  is a noisy signal of all the other agents' signals
- Thus, if  $j$  believes to have received a better signal than everybody else, he will buy, otherwise sell
- This is a standard story:  $J$ -traders trade due to a **difference in opinion** – they think the asset is worth more/less than others (as in Hellwig/Brown-Jennings)

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## Results: Demand period 1

- Market clearing in period 2 together with (2) implies

$$p_2 = \int_0^1 \mathbb{E}[v | \eta_j, \nu, \mu_1, \mu_2] dj - \frac{\gamma}{\tau_v^2} u_2$$

- Rewrite period-1 demand using this:

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma} \left( \underbrace{\mathbb{E} \left[ \int_0^1 \mathbb{E}[v | \eta_j, \nu, \mu_1, \mu_2] dj - \frac{\gamma}{\tau_v^2} u_2 \middle| \eta_i, \nu, \mu_1 \right]}_{\text{2nd-order expectation}} - p_1 \right)$$

- $I$ -trader demand in period 1 is thus a function of a **second-order expectation**: The more  $i$  expects  $J$  traders to value the asset, the more he buys

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## Results

- In the paper, Kondor considers a more general information structure where there is a common factor about which  $I$  and  $J$  both learn.
- He then defines *weakly correlated* information structures in which the common factor is not too important – otherwise we're back in Brown and Jennings [1989] world
- In the above, we have disregarded the common factor, so what we analyzed is automatically a weakly correlated information structure

### Main result

If the information structure is *weakly correlated*, then trading intensity, volume and informational content of prices **increase** in both periods when there is more public info.

- Public signals create trade, due to their effect on second-order beliefs

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## Model 2

### Heterogenous trading horizons

- Timing
  - 1 I-traders and J-traders trade
  - 2 I-traders sell all their holdings to J-traders and quit
  - 3  $v$  is realized and J-traders consume
- Let  $\mu$  be the proportion of J-traders

### Interpretation

- Traders with different trading horizons co-exist in the market
- For instance day-traders and pension savers

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## Model 2: Results

- When  $\mu$  is high, most traders trade with each other: the market is well-integrated
- When  $\mu$  is small, the results of model 2 are close to those of model 1
- But when  $\mu$  is high, public information crowds out private information, and public signals have the usual effect
  - I.e., there will be less disagreement and less trade
- Thus, integration is key to the results (what happens to ST speculation as market becomes more integrated?)

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## Relation to empirics

- In general, the model provides an explanation for trade after public announcements
- Bailey, Karolyi, and Salva [2006] find that price volatility and trading volumes increase after earnings announcements
  - They find that the effect is larger for cross-listed stocks
  - Kondor argues that cross-listing is roughly equivalent to lower market integration: lower  $\mu$

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## Kondor: Conclusion

- Public announcements can affect second-order beliefs, thereby generating trade and increasing price volatility!
- This requires some very specific assumptions on the information structure in the market though...
- The model goes primarily towards explaining some empirical puzzles; not clear whether we should base welfare analysis on it.
- But it should allow us to predict better which stocks will react strongly to announcements

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