Ch 6, ex 5

Make/take fees and bid-ask spreads.

Consider the Parlour model. Trading platforms often charge different fees for market and limit orders.

- Let f_{mo} be the fee per share paid by a market order placer and
- f_{lo} the fee per share for a limit order placer when the limit order executes (there is no entry fee for limit orders).
- Finally let f be the total fee earned by the platform on each trade, $f = f_{mo} + f_{lo}$.

Foucault model: lecture ver

- **Exogenous prices**. Bid and ask prices exogenously given as $A > v > B$
- **Traders**. Arriving trader chooses btw limit or market order (one unit)
  - Limit order valid one period. Choice depends on prob. of limit order being executed, i.e. 'hit' by a market order from the next trader
  - Valuation: $v + y$. y is uniformly distributed on $(-Y, Y)$, unobserved and independent across traders. v is known and common to all.
- **Profits**. Let $P_M^B (P_M^S)$ be prob. of next-period market buy (sell) order

Foucault/(Parlour) model: lecture vs book ver

- Model in lecture differs slightly from model in the book (6.4.1-2):
  - we made A and B exogenous, the book derives them;
  - we let $y_i \sim U[-L, L]$, the book assumes $y_i \in \{-L, L\}$;
  - in 6.4.1 the textbook initially sets up a much more general model, but never actually solves it;
  - in lecture, we implicitly assumed that if LOB is empty on either side, it is filled by MM at same prices. Don't really need to
- For the problem today we stick to the textbook version (6.4.2)

Ch 6, ex 5, part a

(a) Compute bid and ask quotes in equilibrium

How do?

Ch 6, ex 5, part a

(a) Compute bid and ask quotes in equilibrium

How should traders behave in equilibrium?

- If $y_i = L$ then buy
  - indifferent between market buy (if available) and limit buy
- If $y_i = -L$ then sell
  - indifferent between market sell (if available) and limit sell

Ch 6, ex 5, part a

Consider $y_i = L$.

- Profit from market buy is $v + L - A - f_{mo}$.
- Profit from limit buy is $(v + L - B - f_{lo})P_M^S$.
- Indifference $\Rightarrow$ the two are equal. This gives a condition on A, B given $v, L, f_{mo}, f_{lo}, P_M^S$.
- But P_M^S is uncertain – even if next trader has $y_i = -L$, how does he choose between MS and LS?
  - In equilibrium: if $t+1$-trader *can* trade with t-trader, then will always choose so.
  - Idea: limit trader at t can set a price that is ϵ-better for $t+1$ than submitting a limit order. So anything different from the above cannot be an equilibrium.

Ch 6, ex 5, part a

So indifference condition is:

$$v + L - A - f_{mo} = (v + L - B - f_{lo})1/2$$

Same for trader with $y_i = -L$:

$$B - (v - L) - f_{mo} = (A - (v - L) - f_{lo})1/2$$

Solve the two for A, B to get:

$$A = v + \frac{1}{3}(L + f_{lo} - 2f_{mo})$$
$$B = v - \frac{1}{3}(L + f_{lo} - 2f_{mo})$$

Ch 6, ex 5, part b

(b) Show that the bid-ask spread decreases in f_{mo} and increases in f_{lo}. Explain.

Ch 6, ex 5, part b

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$$S = \frac{2}{3}(L + f_{lo} - 2f_{mo})$$

- if limit orders expensive then the price improvement from LO compared to MO (=spread) must be large to offset this cost, make LO competitive with MO
- vice versa for f_{mo}

Ch 6, ex 5, part c

(c) Trading platforms often subsidize traders who submit limit orders. That is, they set $f_{lo} < 0$ and $f_{mo} > 0$, maintaining that this practice ultimately helps to narrow the spread and benefits traders submitting market orders. Holding the total trading fee fixed, is this argument correct?

Ch 6, ex 5, part c

(c) Trading platforms often subsidize traders who submit limit orders. That is, they set $f_{lo} < 0$ and $f_{mo} > 0$, maintaining that this practice ultimately helps to narrow the spread and benefits traders submitting market orders. Holding the total trading fee fixed, is this argument correct?

This does narrow down the nominal spread, but does NOT benefit market traders.

Consider a MB order. Trader pays

$$A + f_{mo} = v + \frac{1}{3}(L + f)$$

which only depends on total f and not on how it is split between f_{lo} and f_{mo}.