

Financial Markets Microstructure

Lecture 9

Limit order book, part 1
Chapter 6.1-6.2 of FPR

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Last time

- The Kyle model helps us analyze market depth:
 - Hence, it tells us something about how adverse selection causes spread to vary with trade size
 - The model has batch clearing instead of the single-unit market of GM
- We can use insights from theory to estimate the importance of different components of the spread
 - Perhaps surprisingly, order costs are by far the largest cost (but estimated on major stocks)
 - Around 19% of trading is informed
 - Adverse selection is stronger for less liquid/small-cap stocks

Homework

- Read the Economist article on the corporate bond market.
Discuss the following questions:
 - 1 How does corporate bond market liquidity differ from the stock market liquidity? Why?
 - 2 Why do investors' liquidity expectations matter?
 - 3 How do investors form their expectations of liquidity?
 - 4 Can we *measure investors' expectations* of liquidity?

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LOB Markets: Liquidity Demand

- Most markets are order-driven these days
 - Some exchanges combine LOBs and dealers
- There is also conceptual convenience in starting with dealer models...
- “Dealer” is an abstraction of the “market”:

Trader interacting with the dealer \approx Trader interacting with the market

So for a **liquidity demander** who submits market orders, it does not really matter who to trade against.

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LOB Markets: Liquidity Providers

But **liquidity provision** works quite differently in LOB markets:

- 1 Traders have a choice between **demanding and supplying liquidity** (market vs limit orders) – so LO should be no worse than MO for trading
- 2 LO give better prices but suffer from execution risk
- 3 Conditional on electing to trade via LO, traders face a **different informational environment** from the dealer (dealer knows more about what's happening in a market)
- 4 Also there's discriminatory pricing: LO execute each at their posted price, while a dealer can clear everything at uniform price.

We look at the two latter issues today; two former next time.

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LOB Markets: Modelling Issues

A limit order is (t, x, p) : submission time t , quantity x , price p .

Modelling LOBs properly is very difficult, many factors to account for:

- 1 Traders' **action set** is large: submit buy/sell LO/MO, how much, choose price if LO
- 2 **Dynamics** is relevant: choose when to submit, when to cancel/resubmit (since LO might not execute)
- 3 **State** of LOB is rich: trader's choice depends on other LOs in the book (though not all of them might be observable)
- 4 Private information affects not just valuation, but **expectations** about LOB evolution...

So we'll look at very simple models that explore different slices of this problem.

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? model

- Begin by looking at ? model
 - (Probably) the first model of LOB markets
 - Does not capture all of traders' choices
 - Consider it to be "one step towards LOB markets from Kyle model".
- Questions simple:
 - What drives the prices? Are they efficient?
 - How are prices different from dealer markets?
 - How is LOB depth determined?

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Continuous model: Limit order book (asset supply)

- **Single asset:** Unknown value v with cdf $G(v)$
- **Limit order book:**
 - Focus on the ask side of the market (limit orders to sell vs market order to buy)
 - Assume a continuum of competitive limit traders, each submits an infinitesimally small order
 - Let $p(q) \equiv$ price of q th unit (increasing in q due to price priority)
 - A market buy order of size q will 'walk up the book' until the final bit of it is cleared at price $p(q)$
 - The entire payment for buying volume q is then $P(q) \equiv \int_0^q p(\tilde{q}) d\tilde{q}$.
 - The average price per unit is $\bar{p}(q) \equiv \frac{P(q)}{q}$ – the (inverse) **supply curve**
- Bliz quiz: in the Kyle model, did the dealer announce **marginal price** $p(q)$ or **average price** $\bar{p}(q)$ schedule?

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Continuous model: Market orders (asset demand)

- **Single market[-order] trader** per period (as in GM/Kyle)
- Abstract view: assume some total utility $U(x|v)$ from buying x units, increasing in v , increasing and concave in x , and $\frac{\partial^2 U(x|v)}{\partial x \partial v} > 0$.
- Trader then **demands quantity x** that solves

$$\max_x \{U(x|v) - P(x)\} \quad \Rightarrow \quad U'(x|v) = P'(x) = p(x)$$

- If trader was perfectly informed, then $U(x|v) = v \cdot x$
- Assume again some superposition of informed and uninformed traders, $U(x|v)$ is average over types
- Abusing notation, $U'(x|v)$ is average/representative trader's marginal valuation for x th unit of the asset
- The cross-derivative implies that $\mathbb{E}[v|U'(x|v) = p]$ for a given x is strictly increasing in p
 - the fact that the trader stopped at a higher marginal price p suggests higher v

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Continuous model: Market makers

- **Limit[-order] traders** are competitive, post limit orders
 - Think of a continuum of traders, each posts a limit order for one (infinitesimally small) unit of the asset
- Limit price $p(q)$ quoted for the q -th unit is relevant (=is traded against) iff

$$x \geq q \iff U'(x) \geq p(q)$$

- Hence, if supply is competitive then in equilibrium,

$$p(q) = \mathbb{E}[v|x \geq q]$$

(as before - price of trade equals the expected value of the traded asset. The conditioning event is the interesting part)

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Continuous model: Equilibrium

The assumed correlation of v and x implies that

$$p(q) = \mathbb{E}[v|x \geq q] > \mathbb{E}[v|x = q] \quad (1)$$

(to compare, in the Kyle model we had **what pricing schedule?** $\bar{p}(q) = \mathbb{E}[v|x = q]$)

Implications

- Thus, market makers (ex post) profit on the sale of the last units
- At small realized trades, $q \simeq 0$, MM always profit
- Even with continuous prices, there is a non-zero **inside spread** between ask and bid prices as the order size goes to zero (contrast with Kyle)
- After a trade of size q , new expected asset value is below $p(q)$.
New asset valuation is $\mu = \mathbb{E}[v|x = q] < \mathbb{E}[v|x \geq q] = p(q)$. Often new limit orders will be posted below $p(q)$ – price reversal.

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Conclusion

- First look into LOB markets using ?.
- Limit traders act in the same capacity as the dealer did before
 - but face different **informational environment**
 - so act differently
 - which leads to different market outcomes
- Competitive limit traders may get positive or negative profit ex post, depending on the order they trade against, but get zero profit on average.

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Next time

- What happens if we restrict prices to discrete ticks?
- Many aspects of market design (tick size, priority rules) are all double-edged swords, swing them carefully
- (maybe) Dynamic analysis: traders can choose between limit and market orders