Financial Markets Microstructure

Lecture 6

Liquidity and Price Dynamics

Chapter 3.4-3.7 of FPR

Egor Starkov

Københavns Unversitet

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What did we do last week?

1. Information and prices
2. Efficiency and markets
3. Glosten and Milgrom: Workhorse model to analyze adverse selection in markets
    - Analysis of what drives the spread
    - Tradeoff between market liquidity and price discovery
    - The model had reasonably good efficiency properties

Today

1. Look at other drivers of the spread
   - Order-processing costs
   - Dealer inventory risk
2. We'll look at how their dynamic effect on prices differ

This lecture:

1. Order-processing costs
2. Inventory risk

What order-processing costs exist?

A liquidity supplier (for instance a dealer) can have a range of different order-processing costs

- Trading fees: charged by exchanges
- Clearing and settlement fees: paid if a central clearinghouse is used to minimize trading risks
- Overheads: back office expenses
- (Dealer rents)

These costs must somehow be compensated by traders, and will therefore enter the spread

How do these costs affect the spread?

- Let $\mu_t \equiv \mathbb{E}[v|\Omega_t]$ be the expectation of v *after* the time-t trade order is observed, and let s_t^a and s_t^b denote the 'half-spreads'
- Hence, μ_{t-1} represents what we now when period t starts
- Then in the GM model we can write prices as

$$a_t = \mu_{t-1} + s_t^a$$
$$b_t = \mu_{t-1} - s_t^b$$

- Assume dealer has order cost γ, and charges this directly to trader:

$$a_t = \mu_{t-1} + \gamma + s_t^a$$
$$b_t = \mu_{t-1} - \gamma - s_t^b$$

How do these costs affect the spread? (2)

- Hence, the new bid-ask spread is

$$S_t = a_t - b_t = 2\gamma + s_t^a + s_t^b$$

- The spread is now made up of order costs (2γ) and adverse selection costs ($s_t^a + s_t^b$)
- Suppose we want to determine whether spread in a given market is due to adverse selection or order costs
  - The instantaneous effect of order costs is similar to that of adverse selection costs
  - But we shall see that the dynamic effect is different

The dynamics of the spread

- As before, let $d_t = 1$ denote a buyer-initiated trade, and $d_t = -1$ a seller-initiated trade
- Also, let $s(d_t)$ be the adverse-selection-related half-spread depending on the trade: $s(1) = s_t^a$ and $s(-1) = s_t^b$
- Then the realized price can be written as

$$p_t = \mu_{t-1} + (s(d_t) + \gamma)d_t$$

- Since $\mu_t = \mu_{t-1} + s(d_t)d_t$, then

$$p_t = \underbrace{\mu_t}_{\text{updated valuation}} + \underbrace{\gamma d_t}_{\text{order cost}}$$

The dynamics of the spread

Then the effect of time-t trade on prices:

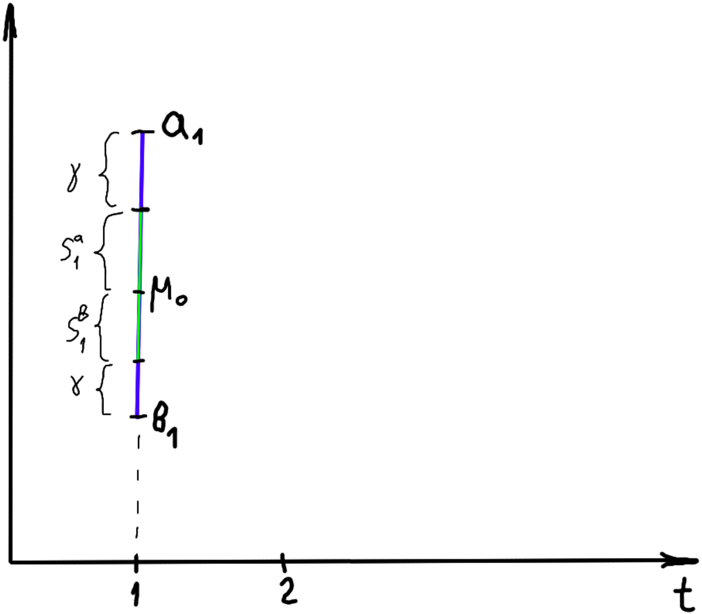
- **short-run**:

$$p_t - \mu_{t-1} = (s(d_t) + \gamma)d_t$$

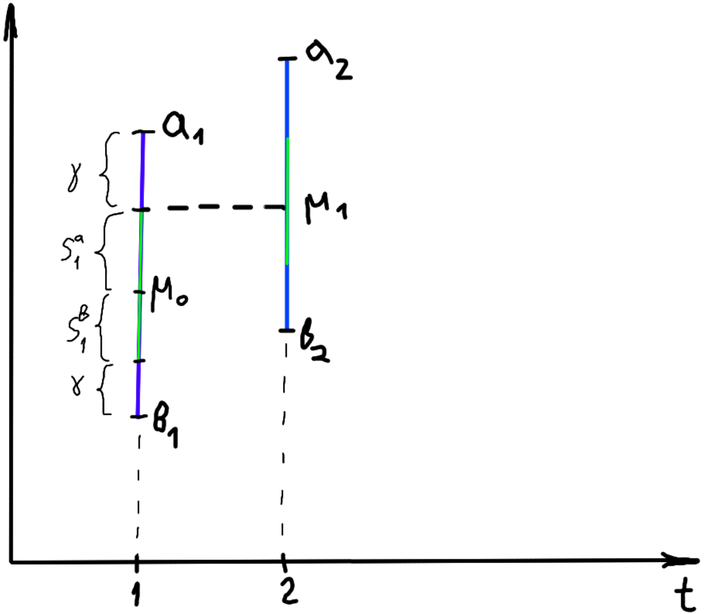
- **long-run**:

$$\begin{aligned}
\mathbb{E}_t[p_{t+s}] - \mu_{t-1} &= \mathbb{E}_t[\mu_{t+s-1} + (s(d_{t+s}) + \gamma)d_{t+s}] - \mu_{t-1} \\
&\approx \mathbb{E}_t[\mu_{t+s-1}] - \mu_{t-1} \\
&= \mu_t - \mu_{t-1} \\
&= s(d_t)d_t
\end{aligned}$$

- so order cost effect on prices is *transient* and is reversed by future trades;
- effect of adverse selection term is *permanent*



a_1
γ
S_1^a
μ_0
S_1^B
γ
B_1
1
2
t



a_2
a_1
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S_1^a
μ_0
S_1^b
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γ
b_1
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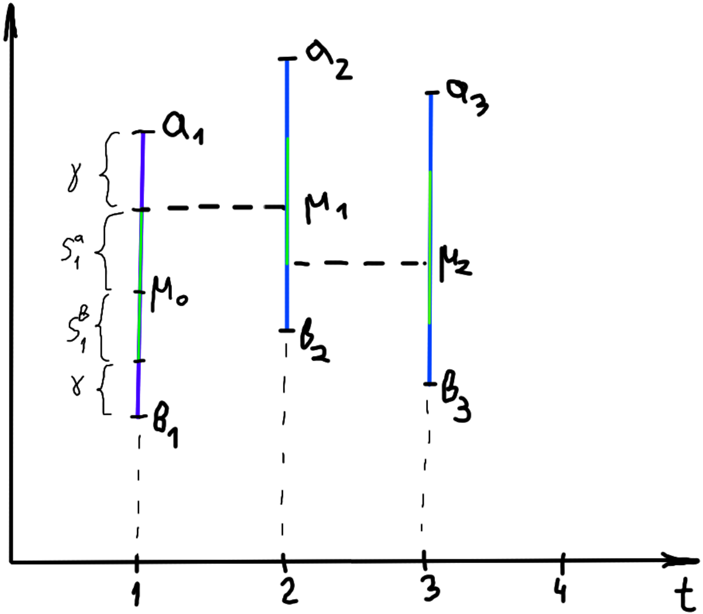

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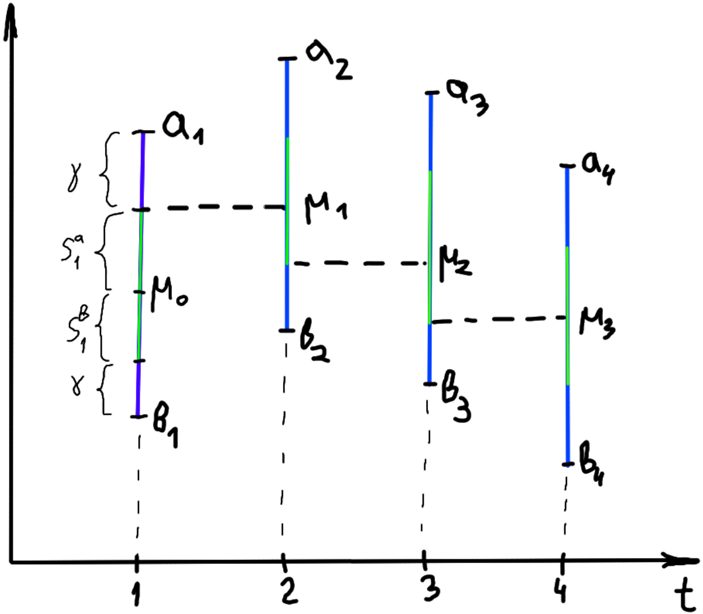

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2. Inventory risk

Inventory risk

- Illiquidity can arise due to dealers' asset inventory cost
  - Holding inventory is risky, so dealers adjust their quotes to unwind any accumulated inventory.

Inventory risk

- Illiquidity can arise due to dealers' asset inventory cost
  - Holding inventory is risky, so dealers adjust their quotes to unwind any accumulated inventory.
- Textbook illustrates this using Stoll (1978) model
  - But this model is from pre-game theory times and has a strange solution method
  - So we will instead extend the Glosten-Milgrom model (extension not in the book)

Glosten-Milgrom model: Inventory risk edition

- Suppose for simplicity there are no speculators ($\pi = 0$)...
- ...but there are public news about asset value: $\mu_{t+1} = \mu_t + \epsilon_t$ (where $\mu_t = \mathbb{E}[v|\Omega_t]$)
- Noise traders behave as usual
- Dealer has some inventory z_t of the stock and c_t of cash
- Dealer is risk averse
  - For concreteness, assume mean-variance preferences over next-period wealth:

    $$U(w_{t+1}) = \mathbb{E}[w_{t+1}] - \frac{\rho}{2}\mathbb{V}(w_{t+1}),$$

    where $w_t = z_t \mu_t + c_t$ and $\rho > 0$ measures risk aversion
  - (equivalent to CARA expected utility preferences when returns are normal)

GM-IRE: Dealer's utility

- As usual, let $\mathbb{E}[\epsilon_t] = 0$, $\mathbb{V}(\epsilon_t) = \sigma^2$, and remember that z_t is the current inventory, and $w_t = z_t \mu_t + c_t$.
- Dealer's utility at the end of period t is

$$\begin{aligned} \text{if no orders:} \quad & U(w_{t+1}|N) = \\ \text{if Buy order:} \quad & U(w_{t+1}|B) = \\ \text{if Sell order:} \quad & U(w_{t+1}|S) = \end{aligned}$$

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- Dealer's utility at the end of period t is

$$
\begin{aligned}
\text{if no orders:} \quad & U(w_{t+1}|N) = z_t \mu_t + c_t - \frac{\rho}{2} z_t^2 \sigma^2 \\
\text{if Buy order:} \quad & U(w_{t+1}|B) = [(z_t - 1)\mu_t + c_t + a_t] - \frac{\rho}{2}(z_t - 1)^2 \sigma^2 \\
\text{if Sell order:} \quad & U(w_{t+1}|S) = [(z_t + 1)\mu_t + c_t - b_t] - \frac{\rho}{2}(z_t + 1)^2 \sigma^2
\end{aligned}
$$

GM-IRE: Quotes

To derive equilibrium quotes, use the zero-profit condition

- i.e., ensure that the dealer's expected utility does not change after trading
- (assume all competing dealers have same inventory z_t)

$$U(w_{t+1}|B) = U(w_{t+1}|N) \quad \Rightarrow \quad a_t = \mu_t - \rho\sigma^2 z_t + \frac{\rho}{2}\sigma^2$$

$$U(w_{t+1}|S) = U(w_{t+1}|N) \quad \Rightarrow \quad b_t = \mu_t - \rho\sigma^2 z_t - \frac{\rho}{2}\sigma^2$$

GM-IRE: Quotes (2)

- Spread is $S_t = \rho\sigma^2$
  - Positive due to dealer's risk-aversion
  - Increasing in risk-aversion coeff ρ and asset value volatility σ^2

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- Spread is $S_t = \rho\sigma^2$
  - Positive due to dealer's risk-aversion
  - Increasing in risk-aversion coeff ρ and asset value volatility σ^2
- Midquote is $m_t = \mu_t - \rho\sigma^2 z_t$
  - Depends on dealer's inventory z_t. Dealer demands risk premium for taking a position in the asset.
  - Effects depend on risk paratements ρ and σ^2
  - To emphasize: prices here are not efficient
  - This inefficiency would in principle also motivate traders to submit the "right" orders – arbitrage! (Not present in this model, since we assumed traders are not strategic and not sensitive to the price.)

Summary

- The spread is driven not only by adverse selection: order costs and inventory risk have an effect as well
    - Price efficiency is lost once these new factors come into play
- How can we figure out which of these factors are more/less important? By their long-term effects! Will talk about that soon.
- But before we go there, next time: what affects market *depth*?

Homework

We said today that inventory risk is priced when the dealer is risk-averse. Risk-aversion is one explanation, but other factors can also contribute to inventory risk. The two following cases explore this issue:

- A big trader was punted off the Nordic power market after failing to meet margin calls (two articles on absalon).
  - How does inventory risk manifest in this story?
  - Explain why such inventory risk can be priced even by risk-neutral agents.
- Negative oil futures prices were registered in 2020 (blog post on absalon or here).
  - Why did it happen? How do negative prices make sense?
  - How does inventory risk manifest in this story?
  - Explain why such inventory risk can be priced even by risk-neutral agents.