Financial Markets Microstructure

Lecture 9

Limit order book, part 1

Chapter 6.1-6.2 of FPR

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Spring 2025

Last time

- The Kyle model helps us analyze market depth:
  - Hence, it tells us something about how adverse selection causes spread to vary with trade size
  - The model has batch clearing instead of the single-unit market of GM
- We can use insights from theory to estimate the importance of different components of the spread
  - Perhaps surprisingly, order costs are by far the largest cost (but estimated on major stocks)
  - Around 19% of trading is informed
  - Adverse selection is stronger for less liquid/small-cap stocks

Homework

- Read the Economist article on the corporate bond market.
  Discuss the following questions:
  1. How does corporate bond market liquidity differ from the stock market liquidity? Why?
  2. Why do investors' liquidity expectations matter?
  3. How do investors form their expectations of liquidity?
  4. Can we *measure investors' expectations* of liquidity?

This lecture:

1. LOB Markets: Introduction

2. Static Analysis: Glosten Model (continuous)

Introduction

- Note: this and the next two lectures draw a bit from the Parlour and Seppi [2008] survey in addition to the textbook.
- Note: the notation will also be a bit different from the book to stay closer to what we used before

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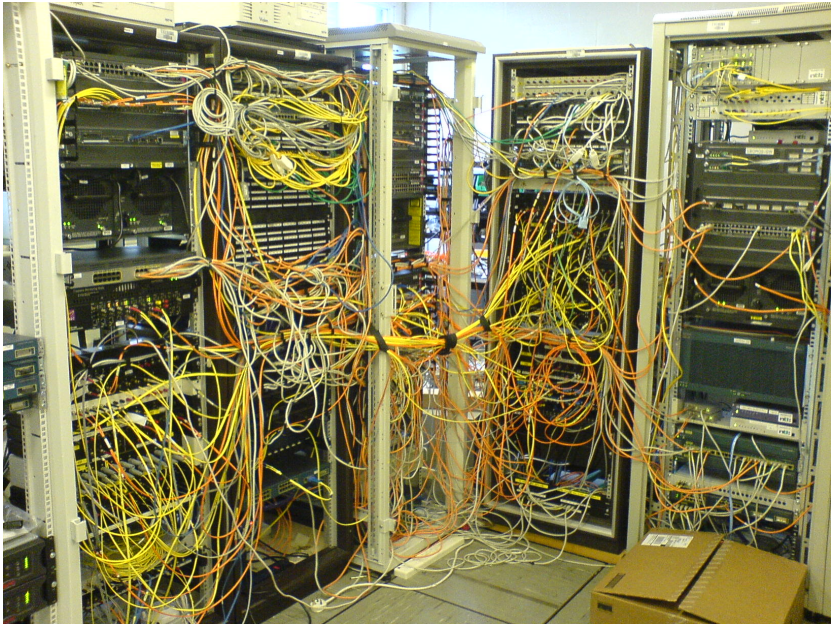
- There's a reason that first models of financial markets (GM and Kyle) explored dealer markets...

Financial market in 1985



Financial market today

LOB Markets: Liquidity Demand

- Most markets are order-driven these days
  - Some exchanges combine LOBs and dealers
- There is also conceptual convenience in starting with dealer models...
- "Dealer" is an abstraction of the "market":

  Trader interacting with the dealer $\approx$ Trader interacting with the market

  So for a **liquidity demander** who submits market orders, it does not really matter who to trade against.

LOB Markets: Liquidity Providers

But liquidity provision works quite differently in LOB markets:

1. Traders have a choice between demanding and supplying liquidity (market vs limit orders) – so LO should be no worse than MO for trading
2. LO give better prices but suffer from execution risk
3. Conditional on electing to trade via LO, traders face a different informational environment from the dealer (dealer knows more about what's happening in a market)
4. Also there's discriminatory pricing: LO execute each at their posted price, while a dealer can clear everything at uniform price.

We look at the two latter issues today; two former next time.

LOB Markets: Modelling Issues

A limit order is (t, x, p): submission time t, quantity x, price p.

Modelling LOBs properly is very difficult, many factors to account for:

1. Traders' action set is large: submit buy/sell LO/MO, how much, choose price if LO
2. Dynamics is relevant: choose when to submit, when to cancel/resubmit (since LO might not execute)
3. State of LOB is rich: trader's choice depends on other LOs in the book (though not all of them might be observable)
4. Private information affects not just valuation, but expectations about LOB evolution...

So we'll look at very simple models that explore different slices of this problem.

This lecture:

1 LOB Markets: Introduction

2 Static Analysis: Glosten Model (continuous)

Glosten [1994] model

- Begin by looking at Glosten [1994] model
  - (Probably) the first model of LOB markets
  - Does not capture all of traders' choices
  - Consider it to be "one step towards LOB markets from Kyle model".
- Questions simple:
  - What drives the prices? Are they efficient?
  - How are prices different from dealer markets?
  - How is LOB depth determined?

Continuous model: Limit order book (asset supply)

- **Single asset**: Unknown value v with cdf $G(v)$
- **Limit order book**:
  - Focus on the ask side of the market (limit orders to sell vs market order to buy)
  - Assume a continuum of competitive limit traders, each submits an infinitesimally small order
  - Let $p(q) \equiv$ price of qth unit (increasing in q due to price priority)
  - A market buy order of size q will 'walk up the book' until the final bit of it is cleared at price $p(q)$
  - The entire payment for buying volume q is then $P(q) \equiv \int_0^q p(\tilde{q})\, d\tilde{q}$.
  - The average price per unit is $\bar{p}(q) \equiv \frac{P(q)}{q}$ – the (inverse) supply curve

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- Bliz quiz: in the Kyle model, did the dealer announce marginal price $p(q)$ or average price $\bar{p}(q)$ schedule?

Continuous model: Market orders (asset demand)

- **Single market[-order] trader** per period (as in GM/Kyle)
- Abstract view: assume some total utility $U(x|v)$ from buying x units, increasing in v, increasing and concave in x, and $\frac{\partial^2 U(x|v)}{\partial x \partial v} > 0$.
- Trader then demands quantity x that solves

$$\max_x \{U(x|v) - P(x)\} \qquad \Rightarrow \qquad U'(x|v) = P'(x) = p(x)$$

  - If trader was perfectly informed, then $U(x|v) = v \cdot x$
  - Assume again some superposition of informed and uninformed traders, $U(x|v)$ is average over types
- Abusing notation, $U'(x|v)$ is average/representative trader's marginal valuation for xth unit of the asset
- The cross-derivative implies that $\mathbb{E}[v|U'(x|v) = p]$ for a given x is strictly increasing in p
  - the fact that the trader stopped at a higher marginal price p suggests higher v

Continuous model: Market makers

- **Limit[-order] traders** are competitive, post limit orders
  - Think of a continuum of traders, each posts a limit order for one (infinitesimally small) unit of the asset
- Limit price $p(q)$ quoted for the q-th unit is relevant (=is traded against) iff

$$x \geq q \iff U'(x) \geq p(q)$$

- Hence, if supply is competitive then in equilibrium,

$$p(q) = \mathbb{E}[v|x \geq q]$$

(as before - price of trade equals the expected value of the traded asset. The conditioning event is the interesting part)

Continuous model: Equilibrium

The assumed correlation of v and x implies that

$$p(q) = \mathbb{E}[v|x \geq q] > \mathbb{E}[v|x = q] \tag{1}$$

(to compare, in the Kyle model we had what pricing schedule?

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Implications

- Thus, market makers (ex post) profit on the sale of the last units
- At small realized trades, $q \simeq 0$, MM always profit
- Even with continuous prices, there is a non-zero inside spread between ask and bid prices as the order size goes to zero (contrast with Kyle)
- After a trade of size q, new expected asset value is below $p(q)$.

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- After a trade of size q, new expected asset value is below $p(q)$.
  New asset valuation is $\mu = \mathbb{E}[v|x = q] < \mathbb{E}[v|x \geq q] = p(q)$. Often new limit orders will be posted below $p(q)$ – price reversal.

Conclusion

- First look into LOB markets using Glosten [1994].
- Limit traders act in the same capacity as the dealer did before
  - but face different informational environment
  - so act differently
  - which leads to different market outcomes
- Competitive limit traders may get positive or negative profit ex post, depending on the order they trade against, but get zero profit on average.

Next time

- What happens if we restrict prices to discrete ticks?
- Many aspects of market design (tick size, priority rules) are all double-edged swords, swing them carefully
- (maybe) Dynamic analysis: traders can choose between limit and market orders

References I

L. R. Glosten. Is the Electronic Open Limit Order Book Inevitable? *The Journal of Finance*, 49 (4):1127–1161, 1994. ISSN 1540-6261. URL https://doi.org/10.1111/j.1540-6261.1994.tb02450.x.

C. A. Parlour and D. J. Seppi. Limit Order Markets: A Survey. In *Handbook of Financial Intermediation and Banking*, pages 63–96. Elsevier, 2008. ISBN 978-0-444-51558-2. doi: 10.1016/B978-044451558-2.50007-6. URL https://linkinghub.elsevier.com/retrieve/pii/B9780444515582500076.