

Financial Markets Microstructure

Exercises after L3

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This lecture:

1 Problems from Lecture 2

Lecture 2

From Lecture 2:

- problem 8 from FRP ch.2 (pp.72-76)
- reproduce graphs from lecture (\approx problems 6-7 from FPR ch.2) – we did that in lecture

Problem 8: Implementation Shortfall

Your client wants to buy q shares in company XYZ at time 0 and hold them until time t . Midprice at time 0 is m_0 . Market is not perfectly liquid, so realized [average] purchase price will be $\bar{p} = m_0 + \lambda k q$, where $k \in [0, 1]$ is the share of the order fulfilled, λ price pressure parameter. Expected midprice at time t is m_t (independent of k).

Choose k to minimize the resulting implementation shortfall.

Problem 8: Implementation Shortfall

Implementation shortfall:

$$\begin{aligned}IS_t &= q(m_t - m_0) - kq(m_t - \bar{p}) \\&= kq(\bar{p} - m_0) + (1 - k)q(m_t - m_0) \\&= \lambda(kq)^2 + (1 - k)q(m_t - m_0).\end{aligned}$$

Problem 8: Implementation Shortfall

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Find min:

$$\begin{aligned}\frac{dIS_t(k)}{dk} &= 2\lambda q^2 k - q(m_t - m_0) = 0 \\ \Leftrightarrow k &= \frac{m_t - m_0}{2\lambda q}\end{aligned}$$

(don't forget to check the second order condition: $\frac{d^2 IS_t(k)}{dk^2} > 0$)

Problem 8: Implementation Shortfall

$$k = \frac{m_t - m_0}{2\lambda q}$$

- Increasing in $m_t - m_0$:
 - if asset price is expected to increase, the opportunity cost (regret) of not buying the asset is high.
- Decreasing in λ :
 - the more sensitive is the price, the less you can buy cheaply enough
- Decreasing in q :
 - (same as above) large orders move prices by more – costlier to fulfill fully.

Problem 8: Implementation Shortfall

- If you were solving this problem in reality, you would not know m_t .
- Instead, treat m_t as your **expectation** of the future price, choose trading strategy to minimize the **expected implementation shortfall**.