Financial Markets Microstructure

Lecture 13

Market Fragmentation

Chapter 7 of FPR

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Last week(s)

- Models of order-driven markets
- (Glosten's model): limit traders provide liquidity in a way similar but not same to dealers
- (Foucault's model): needier traders more likely to use MOs. Also LOB is resilient / self-replenishing
- Market design: measures directed at improving liquidity can backfire by distorting incentives

Today

- Analyze fragmented markets: i.e. multiple markets selling the same asset
- Look at fragmentation costs and benefits
- Compare these across batch markets and LOB markets
- This will give us an opportunity to revisit some of the models we have looked at previously
- Finally, we will look at regulation

This lecture:

1. Fragmentation: general notes
2. Fragmentation in Kyle model
3. Fragmentation in Glosten model

History lesson

- A listing in the US used to confer near-exclusive trading rights
  - For instance, if a stock were NYSE-listed, almost all of the trading would occur on the NYSE
  - Until 1999, most stock options were only traded on the exchange where they were listed
- Many European countries used to require that stocks of their companies are only traded on local exchanges

History lesson

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- Many European countries used to require that stocks of their companies are only traded on local exchanges
- All of that has changed in the past 20 years
  - Today, many (at least high-cap) stocks are cross-listed on many exchanges
  - Even if a stock is not listed on a given exchange, it can be admitted for trading (factsheet from LSE on absalon clarifies the difference between the two)
  - Even listed stocks/assets can be often traded outside the regulated exchanges (dark pools)

Fragmentation and consolidation

- Refer to markets with multiple venues trading the same stock as fragmented (as opposed to consolidated)
- Regulation often tries to create 'virtual consolidation': make fragmented markets act *as if* they were consolidated
  - A central concept in the US is the *National Best Bid and Offer* (NBBO): the highest bid and lowest ask price in (any exchange within) the market at a given time
  - US regulation requires that trade always takes place at the NBBO

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  - US regulation requires that trade always takes place at the NBBO
- EU's ban of countries' "concentration rules" aims for a different kind of consolidation – one where all stocks can be traded on same exchange rather than spread around local exchanges.

Fragmentation and priority violations

- LOB has different kinds of priority: price, visibility, time
- In a fragmented market, no overall coordination mechanism, so all of these rules can be violated:
  - Violation of **visibility priority**. An undisplayed order at a price of 100 might be executed on exchange A even though there are quantities visible at 100 on exchange B
  - Violation of **time priority**. A limit order to buy at a price of 100 that was entered at 10:00 AM on exchange A might be filled before an order to buy at 100 that was entered at 9:30 AM on Exchange B
  - Violation of **price priority**. A limit order to buy at a price 100 on exchange A might be executed even though there is, at the same time, a limit order to buy at a price of 101 on exchange B. Known as a *trade through*

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- Worse price discovery (information is more dispersed)
- Less total liquidity (network effects)

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but also to:

- Lower trading costs (due to competition among exchanges)
- Better price discovery (due to aggregating many different signals instead of one)
- More total liquidity (liq providers face less competition, so more willing to participate)

Will look at some of these and others in greater detail today

Platform competition reduces trading costs

- Competition among exchanges is a significant driver of trading fees and commissions
- Sometimes just the threat of entry is enough
- E.g. case of Dutch stocks:

- before 2003, most of Dutch stocks were traded on Euronext's NSC
- in May 2003, Deutsche Börse and LSE separately announced they wanted to enter this market
- Euronext's order entry fee was 0.3 EUR in Jan 2004...
- ...halved to 0.15 for limit orders on Apr 4, 2004...
- ...suspended for market orders on May 24, 2004 (LSE's EuroSETS launch day)...
- ...and finally waived completely for market and limit orders on Jan 31, 2005 together with reducing execution fees.
- (EuroSETS never really gained much traction)

Brokers' agency

- Traders rarely place their orders directly – they usually go to a broker (bank)
- When market is fragmented, brokers should ideally search all markets for the best price
  - (Not a trivial problem, given the existence of dark pools, crossing networks of unknown depth, and hidden orders and/or iceberg orders on standard exchanges)

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  - Exacerbated by payments brokers can receive from exchanges for routing all or some orders through this exchange
- Regulation (US) aims to solve this via
  - "best execution rules" imposed on brokers
  - no-trade-through/order-protection rules which say order must be routed automatically to trade at NBBO

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Kyle model: refresher

To talk about liquidity and price impact, let us look at the effects of fragmentation within Kyle model.

Recall the baseline model from past lectures:

- Risky asset value $v \sim N(\mu, \sigma_v^2)$
- Insider observes v
    - Places market order x; maximizes $\mathbb{E}[x(v-p)|x]$
- Noise trader demand $u \sim N(0, \sigma_u^2)$
- Market maker observes aggregate order flow $q = x + u$
    - Competitively prices asset at $p = \mathbb{E}[v|q]$ (Recall the difference here to what we did in the LOB model)

Kyle model: refresher (2)

- The insider uses a linear strategy $x = \beta(v - \mu)$
- Market makers observe $q = x + u = \beta(v - \mu) + u$. Then

$$p = \mathbb{E}[v|q] = \mu + \lambda q$$

- Solving the MM's problem we get price-impact parameter

$$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$$

- The insider takes for granted the pricing rule $p = \mu + \lambda q$
  - Expected gain is $x(v - \mu - \lambda x)$ since $\mathbb{E}[u] = 0$
- Solving trader's problem gives $x = \beta(v - \mu)$ where $\beta = 1/(2\lambda)$

Kyle model: refresher (3)

- 'Match the coefficients' $\frac{1}{2\beta} = \lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_u^2}$ to get the equilibrium

$$\frac{1}{\lambda} = 2\beta = 2\frac{\sigma_u}{\sigma_v}$$

- Insider equilibrium trading:

$$x = \frac{\sigma_u}{\sigma_v}(v - \mu)$$

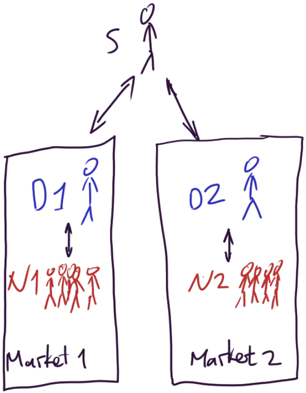
- MM makes zero profit: insider's gain is noise traders' loss

$$\underbrace{\mathbb{E}[x(v-p)]}_{\text{Insider gain}} = \underbrace{\mathbb{E}[u(\lambda u)]}_{\text{Noise loss}} = \lambda\sigma_u^2 = \frac{\sigma_v\sigma_u}{2}$$

Kyle model with a fragmented market

Now fragment the market in this Kyle model. Suppose:

1. Two noise trader groups: $u = u_1 + u_2$, u_i independent and $u_i \sim N(0, \sigma^2_{ui})$
2. Noise traders trade in separate markets; competitive dealers/MMs present in each market (can only see the order in their market); cannot move between markets
3. One speculator trades in both markets (or two speculators trade each in their market – does not really matter).



Fragmented Kyle model: prices

- **Prices**. Since on each market $i = 1, 2$ we must have $p_i = \mathbb{E}[v|q_i]$,

$$\begin{aligned} p_i &= \mu + \lambda_i q_i = \mu + \lambda_i (x_i + u_i) \\ &= \mu + \lambda_i (\beta_i (v - \mu) + u_i) \\ &= \mu + \frac{\sigma_v}{2\sigma_{ui}} \left(\frac{\sigma_{ui}}{\sigma_v} (v - \mu) + u_i \right) \\ &= \mu + \frac{1}{2}(v - \mu) + \frac{\sigma_v}{2\sigma_{ui}} u_i \end{aligned}$$

- On average, price is the same in both markets. But in very short run prices may differ across markets

Fragmented Kyle model: volumes

- **Trading**. Informed trading is given by $x_i = \beta_i(v - \mu)$ with $\beta_i = \frac{\sigma_{ui}}{\sigma_v}$:

$$x_1 + x_2 = (v - \mu)\frac{\sigma_{u1} + \sigma_{u2}}{\sigma_v} > (v - \mu)\frac{\sigma_u}{\sigma_v} = x$$

- More informed trading in total under fragmentation than in consolidated case
- To see this, note:

$$\begin{aligned} \mathbb{V}(u) = \sigma_u^2 = \sigma_{u1}^2 + \sigma_{u2}^2 &= \mathbb{V}(u_1 + u_2) \\ \Rightarrow \sigma_u^2 &< (\sigma_{u1} + \sigma_{u2})^2 \\ \Rightarrow \sigma_u &< \sigma_{u1} + \sigma_{u2} \end{aligned}$$

Fragmented Kyle model: profits

- **Adverse selection costs.** Measure by loss of noise traders:
  - The expected loss of group i is $\sigma_v \sigma_{ui}/2$: Total loss is

$$\frac{\sigma_v(\sigma_{u1} + \sigma_{u2})}{2} > \frac{\sigma_v \sigma_u}{2}$$

- Fragmentation gives greater adverse selection loss
- Of course, also greater profits for informed traders
- In the above, we compare one speculator (per market) under fragmentation to one speculator in the consolidated market.
  If we have different speculators in the two markets who would compete in the consolidated mkt, the effect is even stronger.

Fragmented Kyle model: depth

- **Market depth**. Depth in each market lower than in consolidated market:

$$\lambda = \frac{\sigma_v}{2\sigma_u} < \min\{\lambda_1, \lambda_2\}$$

Fragmented Kyle model: depth

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- What about aggregate depth? Say you want to trade X total.
  - When splitting optimally across two markets $X = x_1 + x_2$, you equalize expected marginal prices: $\mathbb{E}p_1 = \mu + \lambda_1 x_1 = \mu + \lambda_2 x_2 = \mathbb{E}p_2$ (solve a cost minimization problem to confirm this).
  - So $x_1 = \frac{\lambda_2}{\lambda_1+\lambda_2}X$ and $x_2 = \frac{\lambda_1}{\lambda_1+\lambda_2}X$ and trading moves prices in each market to $p_i = \mu + \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}X$ compared to $p = \mu + \lambda X$ in consolidated market. Can verify that $\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2} < \lambda$
  - Fragmented market is deeper in aggregate (that's why the speculation is more aggressive)

Fragmented Kyle model: price discovery

- **Price discovery**: what is the residual variance $\mathbb{V}(v|q)$?
  We used the fact that everything is normal and $\mathbb{E}[q] = 0$ to obtain (see slides on Kyle)

$$v|q \sim \mathcal{N}\left(\mu + \frac{\mathbb{C}(v,q)}{\mathbb{V}(q)}q, \frac{\sigma_v^2}{2}\right)$$

  - This still works in vector form with $\vec{q} = (q_1, q_2)$
  - ...or with $\vec{p} = (p_1, p_2)$, since $p_i = \mu + \lambda_i q_i$

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- Doing the magic, get $\mathbb{E}[v|\vec{p}] = \mu + \frac{2}{3}(p_1 - \mu) + \frac{2}{3}(p_2 - \mu)$ and $\mathbb{V}(v|\vec{p}) = \frac{\sigma_v^2}{3}$
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- Better price discovery in fragmented market once we combine all the information.
- Because have more signals from which to learn about v
  - although again comparing "one speculator in two mkts" vs "one speculator in one mkt". Two competing insiders would yield same price discovery in consolidated market.

More details for the previous slide:

$$
\begin{aligned}
\mathbb{E}[v|p] &= \mathbb{E}[v] + (\mathbb{C}(v,p_1), \mathbb{C}(v,p_2)) \cdot \mathbb{V}^{-1}(p) \cdot (\vec{p} - \mathbb{E}[\vec{p}])^T \,; \\
\mathbb{V}(v|p) &= \mathbb{V}(v) - \mathbb{C}(v,p) \cdot \mathbb{V}^{-1}(p) \cdot \mathbb{C}^T(v,p) \\
p_i &= \mu + \lambda_i \beta_i (v - \mu) + \lambda_i u_i; \qquad \beta_i = \frac{1}{2\lambda_i} = \frac{\sigma_{ui}}{\sigma_v}; \\
\mathbb{C}(v,p_i) &= \lambda_i \beta_i \sigma_v^2 = \frac{\sigma_v^2}{2} \\
\mathbb{V}(p) &= \begin{pmatrix} \mathbb{V}(p_1) & \mathbb{C}(p_1,p_2) \\ \mathbb{C}(p_1,p_2) & \mathbb{V}(p_2) \end{pmatrix} = \begin{pmatrix} \lambda_1^2(\beta_1^2\sigma_v^2 + \sigma_{u1}^2) & \lambda_1\lambda_2\beta_1\beta_2\sigma_v^2 \\ \lambda_1\lambda_2\beta_1\beta_2\sigma_v^2 & \lambda_2^2(\beta_2^2\sigma_v^2 + \sigma_{u2}^2) \end{pmatrix} \\
&= \frac{\sigma_v^2}{4} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\
\Rightarrow \mathbb{V}^{-1}(p) &= \frac{4}{3\sigma_v^2} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}
\end{aligned}
$$

Fragmented Kyle model: liquidity provision

- In the analysis above, traders were chain-bound to their markets
- If many speculators could choose a market to participate in, a jointly optimal thing to do is to join different markets to avoid competing with each other.
  - But note that it is a prisoners' dilemma for them – it is always *individually* optimal to participate in all markets

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- What if noise traders can choose?

Fragmented Kyle model: liquidity provision

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- If many speculators could choose a market to participate in, a jointly optimal thing to do is to join different markets to avoid competing with each other.
  - But note that it is a prisoners' dilemma for them – it is always *individually* optimal to participate in all markets
- What if noise traders can choose?
  - They would go to a deeper market. Depth $\frac{1}{\lambda_i} = \frac{2\sigma_{ui}}{\sigma_v}$.
  - Larger markets would get larger. Liquidity begets liquidity.
  - This is a natural barrier to entry for new trading platforms and source of "payments for order flow" we mentioned

Conclusion: fragmented Kyle model

Adverse selection costs

- Fragmentation is bad for noise traders (welfare)
- If noise traders coordinate, it can be stable that they gather around a less efficient platform
- Fragmentation *may* create extra depth

Aside

- Note that "fragmentation" may also mean time
- Above observations may explain why trade volume is often concentrated at specific times of day (early and late)
- Also an argument for batch trading versus continuous trading

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Glosten model

Competing *limit order books* may also provide better aggregate depth

- To show this, use LOB model of section 6.2 with display cost $C > 0$, tick size $\Delta > 0$, no adverse selection (asset value is μ, only the non-execution risk is relevant)
- Recall: incoming order x, limit sell orders posted at cumulative quantity q and price A satisfy the zero-profit condition

$$0 = \mathbb{P}(x \geq q)[A - \mu] - C,$$

solved by

$$A = \mu + \frac{C}{\mathbb{P}(x \geq q)}. \tag{6.7}$$

Glosten model (2)

Now we make the following assumptions:

- **Fragmentation**. Limit orders are supplied in two markets, I and E ("incumbent" market and "entrant" market); display cost C same in both
  - At single available ask price A, denote cumulative limit sell orders by q^I and q^E
- **Market orders**.
  - Market order is *Buy* with chance $1/2$, and of size $x \sim F(x)$
  - With probability $1-\gamma$, the entire order goes to I (never continues to E even if unfilled)
  - With probability γ, the incoming order is split:
    - With prob. $1/2$, order first goes to I, whatever remains goes to E
    - With prob. $1/2$, order first goes to E, whatever remains goes to I
- **Trade probability**. Equilibrium condition: zero profit for the marginal trader on each market. We thus need to calculate $\mathbb{P}(x_i \geq q^i)$ for each market i.

Glosten model: trading probabilities

- The execution probabilities for the marginal traders are as follows, where $\bar{q} = q^I + q^E$:

$$\mathbb{P}(x_I \geq q^I) = \frac{1}{2}\left[\underbrace{\left(1 - \gamma + \frac{\gamma}{2}\right)(1 - F(q^I))}_{I \text{ is executed first}} + \underbrace{\frac{\gamma}{2}(1 - F(\bar{q}))}_{I \text{ is executed second}}\right] \tag{7.11}$$

$$\mathbb{P}(x_E \geq q^E) = \frac{1}{2}\left[\underbrace{\frac{\gamma}{2}(1 - F(q^E))}_{E \text{ is executed first}} + \underbrace{\frac{\gamma}{2}(1 - F(\bar{q}))}_{E \text{ is executed second}}\right] \tag{7.12}$$

Glosten model: equilibrium

- If both markets active ($q^I > 0$ and $q^E > 0$) then (6.7) holds for both
- Then the probabilities in (7.11) and (7.12) must be identical, implying

$$\left(1 - \gamma + \frac{\gamma}{2}\right)(1 - F(q^I)) = \frac{\gamma}{2}(1 - F(q^E))$$

- When $\gamma < 1$, this implies $F(q^I) > F(q^E)$, so $q^I > q^E$
- Why?

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- When $\gamma < 1$, this implies $F(q^I) > F(q^E)$, so $q^I > q^E$
- Why? Market I has a 'routing advantage', meaning it is more attractive $\rightarrow$ more orders

Glosten model: comparing depth

We can relate q^I, q^E and depth q^C in a consolidated mkt as follows. (Recall $\bar{q} = q^I + q^E$).

- The pricing equation (6.7) also holds with single platform. This implies $\mathbb{P}(x_I \geq q^I) = \mathbb{P}(x \geq q^C)$
- By (7.11): $\mathbb{P}(x_I \geq q^I) = \frac{1}{2}\Big[$ weighted avg. of $1 - F(q^I)$ and $1 - F(\bar{q})\Big]$
- But with a single platform: $\mathbb{P}(x \geq q^C) = \frac{1}{2}[1 - F(q^C)]$
- Hence (since zero-profit cond implies these probabilities are equal):

$$q^I < q^C < q^I + q^E = \bar{q}.$$

- Fragmented LOB market is deeper than consolidated LOB market

Glosten model: conclusion

- Previous slide: fragmentation has positive effect on market depth
- Why? Fragmentation allows for more competition. Fragmentation essentially allows limit orders to *partially* sidestep time priority:
  - When market is consolidated, first limit orders always get executed first
  - But in fragmented market, you can post limit order on another market at same price, and (maybe) get executed first
- As we argued when analyzing the pro-rata rule, removing time priority may lead to more orders in the short run, but there might also be general eqm effects (lower liq provider profit $\rightarrow$ fewer liq providers in the mkt $\rightarrow$ less depth and lower resiliency)
- Section 7.4.3: there is a critical value of γ, below which $q^E = 0$
  - Once again, a barrier to entry for new exchanges

Conclusion

- Fragmentation is ubiquitous
- It is costly for uninformed traders, who would prefer to coordinate on a single market
- Other costs may include less risk sharing and less competition among traders (see book)
- Some benefits are possible (larger depth), depending on setting and trading format

Exercises

- Read the Reuters article on dark markets (on absalon)
- Solve exercise 3 on page 276 (ch.7) on brokers receiving payments for order flow