Financial Markets Microstructure
Lecture 19

Markets and public information

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Previously on FMM

- **High-Frequency Trading** generates informational asymmetries between traders
- If the markets are already reasonably good at matching traders with opportunities, fast trading may be strictly bad for welfare
- While HFTs can provide liquidity, more HFTs does not necessarily improve liquidity
- So it might be optimal to eliminate the speed game, e.g. by moving away from continuous markets to frequent batch auctions

News

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- But we really focused on asymmetric information
  - When private news arrive to insiders, they trade, everyone else learns from trades
- What about symmetric uncertainty? When public news arrive:
  - Glosten-Milgrom/Glosten/Foucault: all orders are repriced, but no trade should take place
  - in reality, a lot of trade after public announcements
- Let's look at how information is aggregated and how this depends on public info!

This lecture:

1. Context 1: Hellwig [1980]
2. Context 2: Brown and Jennings [1989]
3. Kondor [2012]: beliefs
4. Kondor [2012]: trade model

Context: Hellwig [1980]

- A model of how disagreement leads to trading. Forget about liquidity takers/makers.
- Suppose there is a continuum of ex ante symmetric traders $i \in [0,1]$ with CARA prefs:

$$U(W_i) = -e^{-\gamma W_i}, \qquad W_i = (v-p)x_i$$

- One asset, fundamental value $v \sim \mathcal{N}(0, 1/\tau_v)$
- Every trader gets a private signal $\eta_i = v + \epsilon_i$ with $\epsilon_i \sim \mathcal{N}(0, 1/\tau_\epsilon)$ and submits a price-contingent demand schedule $x_i(p)$
- Aggregate supply $u \sim \mathcal{N}(0, \sigma_u^2)$ (provided by noise traders)
- Price p is set to clear the market (by some non-trading market-maker/"the invisible hand"/...)

Hellwig: eqm

- Conjecture a linear eqm, where price is a linear function of v and u (and aggregate $\bar{x} = \int \eta_i di$, which cancels out in the end, conditional on v)
- $v|p, x_i$ is normal with some mean and variance $\Rightarrow$ so is W_i (without noise trades, $v|p$ is degenerate)
- Then trader i's problem is:

$$\begin{aligned}
& \max_{x_i} \left\{ \mathbb{E}\left[-e^{-\gamma W_i} | \eta_i, p \right] \right\} \\
(W_i \text{ normal}) \iff & \max_{x_i} \left\{ \mathbb{E}\left[W_i | \eta_i, p \right] - \frac{\gamma}{2} \mathbb{V}\left[W_i | \eta_i, p \right] \right\} \\
\Rightarrow & x_i(\eta_i, p) = \frac{\mathbb{E}\left[v | \eta_i, p \right] - p}{\gamma \mathbb{V}\left[v | \eta_i, p \right]}, \quad p = \mathbb{E}\Big[\mathbb{E}[v|\eta_i, p] \mid p \Big] - \gamma \mathbb{V}\left[v | \eta_i, p \right] u
\end{aligned}$$

Hellwig: conclusion

- $\mathbb{E}[v|\eta_i, p] = a\eta_i + bp$ for $a, b > 0$
- Traders respond to private signals: high $\eta_i \Rightarrow$ buy, low $\eta_i \Rightarrow$ sell
- This is because i believes that when $\eta_i > p$, this might be because high supply u depressed price, hence a purchase is justified (traders disagree about the fundamental value v)

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- What if the traders care not about v, but the resale value instead?

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Brown and Jennings [1989]

- Now consider the same economy but in dynamics: there are two periods $t = 1, 2$, two populations of traders with CARA preferences:
  - "early traders" $i \in [0, 1]$;
  - "late traders" $j \in [0, 1]$;
  - early traders need to offload their asset holdings to late traders:

$$W_i = (p_2 - p_1)x_i, \qquad W_j = (v - p_2)x_j$$

- Private signals $\eta_i = v + \epsilon_i$, $\eta_j = v + \epsilon_j$
- Asset aggregate supply u_1, u_2 i.i.d. normal in the two periods

Brown-Jennings: eqm

- In $t = 2$, same as Hellwig:

$$x_j(\eta_j, p_2, p_1) = \frac{\mathbb{E}\left[v|\eta_j, p_2, p_1\right] - p_2}{\gamma \mathbb{V}\left[v|\eta_j, p_2, p_1\right]}, \quad p_2 = \mathbb{E}\Big[\mathbb{E}[v|\eta_j, p_2, p_1] \mid p_2, p_1\Big] - \gamma \mathbb{V}\left[v|\eta_j, p_2, p_1\right] u_2$$

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- In $t = 1$, instead of $(v|\eta_i, p_1)$, traders now care about $(p_2|\eta_i, p_1)$:

$$x_i(\eta_i, p_1) = \frac{\mathbb{E}\left[p_2|\eta_i, p_1\right] - p_1}{\gamma \mathbb{V}\left[p_2|\eta_i, p_1\right]}$$

i.e., their demand depends on $\mathbb{E}\left[\mathbb{E}[v|\eta_j, p_2, p_1]|\eta_i, p_1\right]$ – their expectation of later investors' expectation of v.

Brown-Jennings: conclusions

- Second-order beliefs are important for trading volumes in dynamic settings.
- In this specific model, early traders have no reason to believe that late traders' estimate differs from their own:

$$\mathbb{E}\left[\mathbb{E}[v|\eta_j, p_2, p_1]|\eta_i, p_1\right] = \mathbb{E}\left[v|\eta_i, p_1\right],$$

  hence early traders only trade (amongst themselves) for the same reason as before – they disagree about the resale value.

Brown-Jennings: public signals

- Revealing a public signal ν about v at $t = 1$ would make traders agree more (they would put less weight on η_i), hence there would be less trade at $t = 1$; the less trade, the more informative ν is.
- But in reality, there is a lot of trading when public news are revealed (Bailey, Karolyi, and Salva [2006]). **Potential explanations** include:
  - Announcements are made up of public and private signals
    (in the presence of HFTs, public news $\approx$ HFTs' private news)
  - Or traders have heterogeneous priors and can therefore 'agree to disagree'; announcements then can amplify or mitigate these initial disagreements and so generate trade.
- Turns out, there's another explanation: if you craft a more elaborate information structure, you can generate disagreement from public news!

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Kondor [2012]: Example

- Two groups of traders again, I and J
- Fundamental value has two components: $v = v_I + v_J$
- I-trader signal: $\eta_i = v_I + \epsilon^i$
- J-trader signal: $\eta_j = v_J + \epsilon^j$
- Public signal: $\nu = v + \epsilon^p$
- Suppose $v_I, v_J, \epsilon^i, \epsilon^j, \epsilon^p$ are independent and normal with zero mean

Main features: public signal about all of v, private signals about *different aspects* of the fundamentals, v_I and v_J.

Example: Before public announcement

No public news (ν not observed)

- Traders' beliefs about v are

$$\mathbb{E}(v|\eta_j) = \mathbb{E}(v_J|\eta_j) + \mathbb{E}(v_I|\eta_j) = a_J\eta_j + 0, a_J > 0$$
$$\mathbb{E}(v|\eta_i) = \mathbb{E}(v_J|\eta_i) + \mathbb{E}(v_I|\eta_i) = a_I\eta_i + 0, a_I > 0$$

- I-trader's second-order belief is

$$\mathbb{E}(\mathbb{E}(v|\eta_j)|\eta_i) = \mathbb{E}(a_J\eta_j|\eta_i) = 0,$$

i.e., $\mathbb{E}(\mathbb{E}(v|\eta_j)|\eta_i) = \mathbb{E}(v)$ because η_i and η_j are independent

Example: With public announcement

Given public signal ν

- Traders' beliefs about v are

$$\mathbb{E}(v|\eta_j,\nu) = \mathbb{E}(v_J|\eta_j,\nu) + \mathbb{E}(v_I|\eta_j,\nu) = (b_J\eta_j + c_J\nu) + x_J\nu,$$
$$\mathbb{E}(v|\eta_i,\nu) = \mathbb{E}(v_J|\eta_i,\nu) + \mathbb{E}(v_I|\eta_i,\nu) = (b_I\eta_i + c_I\nu) + x_I\nu,$$

  where $b_k, c_k, x_k > 0$ and $b_k < a_k$.
  First-order beliefs of i-traders converge due to public signal

Example: With public announcement

Given public signal ν

- Traders' beliefs about v are

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  where $b_k, c_k, x_k > 0$ and $b_k < a_k$.
  First-order beliefs of i-traders converge due to public signal

- But I-agent's second-order belief is

$$\begin{aligned}\mathbb{E}(\mathbb{E}(v|\eta_j,\nu)|\eta_i,\nu) &= b_J\mathbb{E}(\eta_j|\eta_i,\nu) + (c_J + x_J)\nu,\\ &= b_J(e\nu - f\eta_i) + (c_J + x_J)\nu\end{aligned}$$

  where $e, f > 0$.

Example: Conclusion

- $\mathbb{E}(\mathbb{E}(v|\eta_j,\nu)|\eta_i,\nu)$ is decreasing in η_i and the weight on η_i *increases* with the precision of ν
- i.e., second-order beliefs diverge among *i*-traders: the more precise ν is, the less *I*-traders agree about the resale value of the asset $\Rightarrow$ more trade among *i*!
- This disagreement generates trade after public signals.
- The remainder of the slides presents the Kondor's trading model and derivations in *slightly* more detail.

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Kondor [2012]: Full(er) Model

- **Timing**:
  1. I-traders observe their information and trade
  2. I-traders liquidate all their positions and sell to J-traders
  3. v (distributed as before) is realized and J-traders consume asset
- **Traders**: Price takers, $i, j \sim U(0,1)$, demand $x_i(p_t)$, util $u(W_i) = -e^{-\gamma W_i}$ and wealth

$$W_I = (p_2 - p_1)x_I; \qquad W_J = (v - p_2)x_J.$$

- **Supply**: Time-t asset supply u_t (from noise traders): $u_1 \sim \mathcal{N}(0, 1/\delta_1^2)$, $u_2 \sim \mathcal{N}(0, 1/\delta_2^2)$ $(u_2 \equiv u_1 + \Delta u_2)$

Analysis: Trader maximization

- **Random supply**: Implies that prices are not completely informative.
- **I traders**: Solve

$$\max_{x_I} \mathbb{E}\left[-e^{-\gamma W_I} | \eta_i, \nu, p_1\right]$$

  - CARA utility and normal distributions $\Rightarrow$ can rewrite I traders' problem as

$$\max_{x_I} \left\{\mathbb{E}\left[W_I | \eta_i, \nu, p_1\right] - \frac{\gamma}{2}\mathbb{V}\left[W_I | \eta_i, \nu, p_1\right]\right\}$$

- **J traders**: Solve

$$\max_{x_J} \left\{\mathbb{E}\left[W_J | \eta_j, \nu, p_1, p_2\right] - \frac{\gamma}{2}\mathbb{V}\left[W_J | \eta_j, \nu, p_1, p_2\right]\right\}$$

Analysis: Trader maximization (2)

- Taking the FOC and solving for the demands we get

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma}(\mathbb{E}[p_2|\eta_i, \nu, p_1] - p_1), \tag{1}$$

$$x_{2,j}^* = \frac{\tau_v^2}{\gamma}(\mathbb{E}[v|\eta_j, \nu, p_1, p_2] - p_2), \tag{2}$$

where $\tau_{p_2}^2 = 1/\mathbb{V}(p_2|\eta_i, \nu, p_1)$ and $\tau_v^2 = \mathbb{V}(v|\eta_j, \nu, p_1, p_2)$

- In order to calculate expectations, need to make a conjecture about prices

Analysis: Linear prices and price signals

- **Equilibrium**: Look for equilibrium with linear price function/demand

$$p_1 = \frac{1}{e_1}\left[a_1 v_I + c_1\nu - u_1\right] \tag{3}$$

$$p_2 = \frac{1}{e_2}\left[b_2 v_J + c_2\nu + \eta_2\mu_1 - u_2\right], \tag{4}$$

  for some $a_1, b_2, c_1, c_2, e_1, e_2, \eta_2$, where μ_1 is the price signal of v_I

- **Price signal**: This tells us the information contained in prices:

$$\mu_1 \equiv \mathbb{E}[v_I | p_1, \nu] = \frac{e_1 p_1 - c_1\nu}{a_1} = v_I - \frac{1}{a_1}u_1; \tag{5}$$

$$\mu_2 \equiv \mathbb{E}[v_J | p_1, p_2, \nu] = \frac{e_2 p_2 - c_2\nu - \eta_2\mu_1}{b_2} = v_J - \frac{1}{b_2}u_2. \tag{6}$$

Analysis: Reformulating in terms of price signals

- **Rewrite expectation**. All variables jointly normal $\rightarrow$ linear expressions

$$\mathbb{E}[p_2 | \eta_i, \nu, p_1] = a_1^e \eta_i + b_1^e \nu + c_1^e \mu_1; \tag{7}$$

$$\mathbb{E}[v | \eta_j, \nu, p_1, p_2] = a_2^e \eta_j + b_2^e \nu + c_2^e \mu_1 + d_2^e \mu_2. \tag{8}$$

- **Rewrite FOC**. Plugging (7) and (8) into (1) and (2) we get

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma}(a_1^e \eta_i + b_1^e \nu + c_1^e \mu_1 - p_1); \tag{9}$$

$$x_{2,j}^* = \frac{\tau_v^2}{\gamma}(a_2^e \eta_j + b_2^e \nu + c_2^e \mu_1 + d_2^e \mu_2 - p_2). \tag{10}$$

- **Market clearing**: $u_1 = \int_0^1 x_{1,i}^* di$ and $u_2 = \int_0^1 x_{2,j}^* dj$ determine p_1 and p_2 resp.

Analysis: Equilibrium

- **Matching coefficients**: From market clearing, can show that p_t is linear function as conjectured
- **Equilibrium demand**: Matching up all the coefficients, we can then show that

$$x^*_{1,i} = a_1\eta_i + c_1\nu - e_1 p_1; \tag{11}$$
$$x^*_{2,j} = b_2\eta_j + c_2\nu + \eta_2\mu_1 - e_2 p_2. \tag{12}$$

- Demand is increasing in private signal (η_i/η_j), in public signal (ν), in price signal (μ_1), and decreasing in price (p_t) (recall that traders are price takers)

Results: Demand period 2

- Let's look at what drives the agents' demands. Rewrite period-2 demand as

$$x_{2,j}^{*} = b_2(\eta_j - \mu_2) \tag{13}$$

- Notice that η_j is j's private signal and μ_2 is a noisy signal of all the other agents' signals
- Thus, if j believes to have received a better signal than everybody else, he will buy, otherwise sell
- This is a standard story: J-traders trade due to a **difference in opinion** – they think the asset is worth more/less than others (as in Hellwig/Brown-Jennings)

Results: Demand period 1

- Market clearing in period 2 together with (2) implies

$$p_2 = \int_0^1 \mathbb{E}[v|\eta_j, \nu, \mu_1, \mu_2]dj - \frac{\gamma}{\tau_v^2} u_2$$

- Rewrite period-1 demand using this:

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma} \left(\underbrace{\mathbb{E}\left[\int_0^1 \mathbb{E}[v \mid \eta_j, \nu, \mu_1, \mu_2]dj - \frac{\gamma}{\tau_v^2} u_2 \,\middle|\, \eta_i, \nu, \mu_1 \right]}_{\text{2nd order expectation}} - p_1 \right)$$

- I-trader demand in period 1 is thus a function of a **second-order expectation**: The more i expects J traders to value the asset, the more he buys

Results

- In the paper, Kondor considers a more general information structure where there is a common factor about which I and J both learn.
- He then defines *weakly correlated* information structures in which the common factor is not too important
- In the above, we have disregarded the common factor, so what we analyzed is automatically a weakly correlated information structure

Main result

If the information structure is *weakly correlated*, then trading intensity, volume and informational content of prices increase in both periods when there is more public info.

- Public signals create trade, due to their effect on second-order beliefs

Model 2

Heterogenous trading horizons

- Timing
  1. I-traders and J-traders trade
  2. I-traders sell all their holdings to J-traders
  3. v is realized and J-traders consume
- Let μ be the proportion of J-traders

Interpretation

- Traders with different trading horizons co-exist in the market
- For instance day-traders and pension savers

Model 2: Results

- When μ is high, most traders trade with each other: the market is well-integrated
- When μ is small, the results of model 1 are close to those of model 2
- But when μ is high, public information crowds out private information, and public signals have the usual effect
  - I.e., there will be less disagreement and less trade
- Thus, integration is key to the results (what happens to ST speculation as market becomes more integrated?)

Relation to empirics

- In general, the model provides an explanation for trade after public announcements
- Bailey, Karolyi, and Salva [2006] find that price volatility and trading volumes increase after earnings announcements
    - They find that the effect is larger for cross-listed stock
- Kondor argues that cross-listing is roughly equivalent to lower market integration: lower μ

Kondor: Conclusion

- Public announcements can affect second-order beliefs, thereby generating trade and increasing price volatility!
- This requires some very specific assumptions on the information structure in the market though...
- The model goes primarily towards explaining some empirical puzzles; not clear whether we should base welfare analysis on it.
- But it should allow us to predict better which stocks will react strongly to announcements

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