Financial Markets Microstructure
Lecture 16

Value of Liquidity
Chapter 9 of FPR

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Previously on FMM

- Transparency mostly reallocates welfare across market participants
  - Uninformed traders benefit from T, so T helps liquidity
  - Insiders may lose, so T worsens price discovery
  - Dealers/exchanges may win or lose
- But transparency may also impede risk sharing, foster collusion, and have adverse effects when it is asymmetrically distributed

Today: value of liquidity

- So far we looked at how illiquidity makes asset's trade price deviate from its fundamental value
- But illiquidity may itself affect the asset *value*
- Case study – U.S. Treasury Notes and Bills: (Amihud and Mendelson [1991])
  - notes are long-term (2-10y), bills are short-term ($<$ 12m) US govt loans
  - differ only in terms – so soon-to-mature notes are equivalent to bills
  - but notes trade at a discount relative to bills (i.e., offer higher returns) (as of 1991)
  - why? Notes are less liquid (larger spread and brokerage fees). Why less liquid though?

Value of liquidity

- Why does liquidity affect asset value?
- Intuitively, an illiquid asset is costlier to transact
  - Traders take into account transactions costs
  - Require a return that compensates for the cost
  - Liquidity premium: less liquid assets trade at lower prices
- Liquidity need not be constant over time
  - If illiquidity rises, asset price falls
  - If future liquidity is random, this is a risk factor
  - Liquidity risk may be priced

This lecture:

1. Toy model
2. Clientelle effects
3. Liquidity risk
4. Arbitrage

Liquidity premium (Amihud and Mendelson [1986])

- Before, traders cared only about fundamental value. In this model they care about resale value.
- Consider an asset with *constant* relative spread, $s = (a_t - b_t)/m_t$, but fluctuating midprice m_t
  - Note that
$$a_t = m_t \left(1 + \frac{s}{2}\right) \text{ and } b_t = m_t \left(1 - \frac{s}{2}\right)$$
- Consider a trader who plans to:
  1. buy at t, at the respective ask price a_t,
  2. hold the asset for h periods, and sell at b_{t+h}.

To simplify, suppose the asset pays no dividends.

Deriving the premium: risk-adjusted return

- Let r denote the risk-adjusted real return per period required by the market. What? Then

$$a_t = \frac{\mathbb{E}(b_{t+h})}{(1+r)^h}, \qquad \Rightarrow m_t = \frac{\mathbb{E}(m_{t+h})}{(1+r)^h} \times \frac{1-\frac{s}{2}}{1+\frac{s}{2}}$$

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- If we estimate the required return r using mid-quotes, there is a bias due to illiquidity.
- Let R be the nominal return rate, estimated from the midquotes:

$$m_t = \frac{\mathbb{E}(m_{t+h})}{(1+R)^h}$$

The observed R is different from r!

Deriving the premium: approximation

- Thus, we have

$$(1+R)^h = (1+r)^h \times \frac{1+\frac{s}{2}}{1-\frac{s}{2}} \tag{9.5}$$

Thus: (see next slide for derivation)

$$R \simeq r + \frac{s}{h}$$

- Essentially, the asset's return needs to be higher by s/h in order to compensate for the liquidity cost
  - The difference $R - r$ is a liquidity premium
  - Take h as representative trader's holding period for asset

Appendix on the approximation

- To get the approximation of the previous slide, we must use the approximation $\ln(1+x) \simeq x$ for small x
- Recall that $\ln x^h = h \ln x$
- So taking logs of (9.5) we get

$$h \ln(1+R) = h \ln(1+r) + \ln\left(1+\frac{s}{2}\right) - \ln\left(1-\frac{s}{2}\right)$$

and assuming r, R, and s are small we apply the approximation

$$hR \simeq hr + \frac{s}{2} - \left(-\frac{s}{2}\right)$$

- Rearranging we get the result

Discussion

- Note: what about a round-trip starting from a sale?
  - From $b_t = \mathbb{E}(a_{t+h})/(1+r)^h$ we get $R \simeq r - s/h$ (and then a round-trip starting from a buy is strictly lossy)
  - If $R = r + s/h$, then a round-trip starting from a sale is strictly lossy
- Most assets most of the time are in positive aggregate supply – i.e., buyers have the bargaining power and can demand a positive liquidity premium ($R = r + s/h$).
- Empirical evidence confirms *positive* liquidity premium for stocks, bonds
- More general model and empirics in Bongaerts, De Jong, and Driessen [2011]

This lecture:

Clientelle effects

- We obtained $R = r + s/h$ in our toy model
- In reality investors differ in h, expected holding period
- Consider a toy extension of our toy model, with:
  - Two types of investors with $h_1 < h_2$
  - Two assets with $s_1 < s_2$

Clientelle effects

- Suppose in eqm h_1-investors trade in s_1-asset and h_2-investors trade in s_2-asset
- For this to be an eqm, need $R_1 - s_1/h_1 \geq R_2 - s_2/h_1$ and $R_2 - s_2/h_2 \geq R_1 - s_1/h_2$
- The two conditions are equivalent to

$$\frac{1}{h_2} \leq \frac{R_2 - R_1}{s_2 - s_1} \leq \frac{1}{h_1} \tag{9.10}$$

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- There exist R_1, R_2 (and r) which solve this so all ok
  - There would not be a solution if we assumed the opposite kind of separation
  - We also cannot have both groups indifferent between both assets (would need two equalities in 9.10)

Clientelle effects: discussion

- Some investors specialize in illiquid assets / hope to earn the liquidity premium
  - Should in equilibrium be those who trade less frequently
- (would this explain the case of Treasury Bills vs Bonds?)
- Note: more adverse selection implies larger spread, hence attracts traders with large h

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- We assumed that h are fixed, but all the same logic applies if h is random (e.g., traders randomly get liquidity shocks).
- Clientelle effects would then apply whenever different groups of traders have different *distributions* of h.

This lecture:

Liquidity risk

- IRL, spread s randomly fluctuates over time
- Further, liquidity of any given asset may be arbitrarily correlated with that of other assets or the whole market
- These are risk factors which can also be priced
- Use the Liquidity CAPM model of Acharya and Pedersen [2005]

reminder: regular CAPM

- The standard CAPM postulates that return r_j on asset j is governed by the risk-free rate r_f and a risk premium, which depends on the correlation of r_j with the market return r_M:

$$\mathbb{E}[r_j] = r_f + \beta_j \left[\mathbb{E}[r_M] - r_f\right]$$

$$\text{with } \beta_j = \frac{\mathbb{C}(r_j, r_M)}{\mathbb{V}(r_M)}$$

- In particular, only *systematic* risk enters asset price
- *Idiosyncratic* risk of the asset can be diversified away

Liquidity CAPM

- Investors care for net return $r = R - s$ where s now denotes the liquidity premium
  - Let f denote risk-free, M the market
- Plugging these into the CAPM equation, we get

$$\mathbb{E}[R_j - s_j] = r_f + \lambda_M \beta_j$$

where $\lambda_M = \mathbb{E}[R_M - s_M] - r_f$ is the risk premium and

$$\beta_j = \frac{\mathbb{C}(R_j - s_j, R_M - s_M)}{\mathbb{V}(R_M - s_M)}$$

- Expand $\mathbb{C}(R_j - s_j, R_M - s_M)$:

$$= \mathbb{C}(R_j, R_M) + \mathbb{C}(s_j, s_M) - \mathbb{C}(R_j, s_M) - \mathbb{C}(s_j, R_M)$$

to get $\beta_j = \beta_{1j} + \beta_j^{liq} = \beta_{1j} + \beta_{2j} - \beta_{3j} - \beta_{4j}$ with

$$\beta_{1j} = \frac{\mathbb{C}(R_j, R_M)}{\mathbb{V}(R_M - s_M)} \quad : \text{ ordinary } \beta$$

$$\beta_{2j} = \frac{\mathbb{C}(s_j, s_M)}{\mathbb{V}(R_M - s_M)} \quad : \text{ hedge liquidity with liquidity}$$

$$\beta_{3j} = \frac{\mathbb{C}(R_j, s_M)}{\mathbb{V}(R_M - s_M)} \quad : \text{ hedge liquidity with returns}$$

$$\beta_{4j} = \frac{\mathbb{C}(s_j, R_M)}{\mathbb{V}(R_M - s_M)} \quad : \text{ hedge returns with liquidity}$$

Liquidity risk: alternative model

- Earlier in the course we derived the spread given the asset return/value (Glosten-Milgrom)
- Earlier today we did the opposite: derived the required return keeping the spread fixed (Amihud-Mendelson)
- Duffie, Gârleanu, and Pedersen [2005, 2007] do both simultaneously! In their model:
  - Traders are randomly hit with liquidity shocks, so want to buy/sell asset over time depending on their current situation
  - But may not always find a trade – can get stuck with asset when shocked/without asset when not shocked
  - This liquidity risk enters the equilibrium asset price
  - (And the spread is driven by dealers' market power)
- Click here if you want to see the model (you do not need to know it for the exam)

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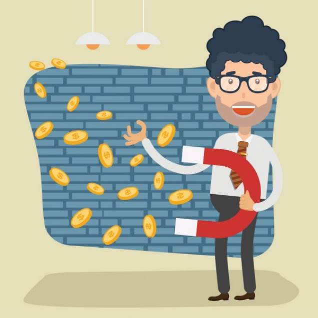
Arbitrage

- Main tenet of economics and finance: no arbitrage
  - Assets that generate same cash flows must cost the same
  - If arbitrage is possible, it is immediately exploited and then there is no more arbitrage.



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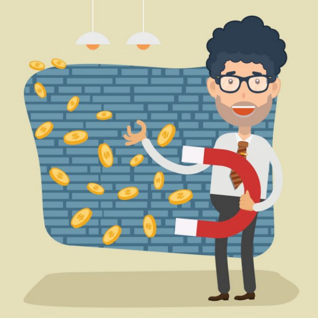
- Bliz quiz: how does this relate to our situation? (think bills-vs-bonds)
  1. Arbitrage opportunities exist because arbitrage is costly to exercise
  2. Arbitrage opportunities exist because all of economics (except our course) is wrong
  3. No arbitrage opportunities exist



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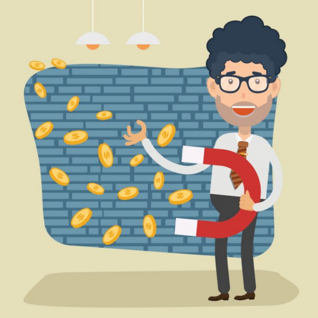
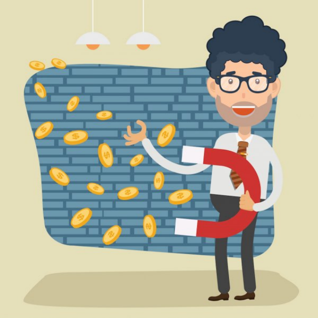
  1. Arbitrage opportunities exist because arbitrage is costly to exercise
     $\uparrow$ textbook answer
  2. Arbitrage opportunities exist because all of economics (except our course) is wrong
  3. No arbitrage opportunities exist $\leftarrow$ my answer



Arbitrage

- Main tenet of economics and finance: no arbitrage
  - Assets that generate same cash flows must cost the same
  - If arbitrage is possible, it is immediately exploited and then there is no more arbitrage.
- Textbook spends an obscene amount of paper arguing why arbitrage cannot be realized in our case (assets with same cash flows but different liquidity)
  - Gist: arbitraging is itself a costly activity (due to leverage and short-selling constraints)
- But in what we saw, there are, strictly speaking, no arbitrage opportunities
  - Arbitrageurs are subject to all the same liquidity costs
  - So in the toy model we saw, it looks as if there is arbitrage, but there are no actual opportunities
  - In the end, it's all about semantics and how you define "arbitrage"



Conclusion

- Empirically the authors find evidence on both a liquidity premium and a liquidity risk premium on stocks
- Further, overall market liquidity may vanish at crisis times when asset prices drop rapidly
  - Important risk for investors, especially speculators
  - Financial institutions are required to hold robustly liquid assets
  - In general, risky positions require costly collateral, margins

Exercise

Ex.1 from ch.9 (p.347)

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Required return

- We have been living in a world with only one asset. In reality, assets “compete” for investors’ attention.
- In market equilibrium, risk-adjusted returns are equalized across assets.
- The resulting “market” return r is what investors can get by investing in any asset, and any new asset must generate return (at least) r to attract funds.

Back

Duffie, Gârleanu, and Pedersen [2005, 2007] Model

- One asset:
  - Pays dividends to its holders each period
  - Traders can hold one or zero units of the asset
  - The alternative is a bank which pays interest r
  - Assume the asset is supplied to fraction $q < 1/2$ of the population
- Unit mass (continuum) of traders, each has either high or low value for the dividend: $j = h, l$
  - If $j = h$, value today's dividend at 1 (high value traders); if $j = l$ then value dividend at $1 - c$ (low value traders), where $c \in (0, 1)$
  - Every period, fraction ψ of traders switch (from h to l or vice versa)
  - (In the long run, half the traders have high value)
  - Switchers would like to trade the asset (h to l $\rightarrow$ want to sell; l to h $\rightarrow$ want to buy)
  - Those willing to trade search for a dealer, find one with probability $\phi < 1$

DGP Model (2)

- Exact timing within a period:
  1. investor receives dividend payoff (\$1 or \$$(1-c)$)
  2. valuation changes w.p. ψ
  3. if needed, trader looks for a dealer; a match happens w.p. ϕ
- Dealers have some bargaining power because they are hard to find
  - Rejecting a dealer's quote means the trader has to wait one period for a chance to meet another dealer – costly (and risky) for a trader
  - Use standard notation: dealers quote ask a and bid b; spread is $S = a - b$; midprice is $\mu = (a + b)/2$
- Look for a stationary equilibrium with a and b constant over time, and all traders with trading needs ("h without an asset" and "l with an asset") looking for a dealer, and agreeing to trade at a and b respectively iff it is optimal for them (may mix).

Solving the DGP model

- Since $q < 1/2$, not all buyers get to buy (buyers must be indifferent $\iff$ dealers have all market power)
- Denote $\bar{a} =$ max price that a buyer will pay, and $\bar{b} =$ min price that a seller will accept
- Buy side: limited supply $\rightarrow$ mixed equilibrium. Set $a = \bar{a}$ and set buy probability conditional on finding dealer (can choose this since buyers are indifferent) to

$$p^B = \phi \cdot \frac{\pi_S}{\pi_B},$$

  π_S: fraction willing to sell; π_B: fraction willing to buy
- Sell side: no restraint, sell with prob. 1, bargain with the dealer over surplus:

$$b = z\bar{b} + (1 - z)\mu.$$

Solving the DGP model (2)

- Method: identify $\bar{a}$ and $\bar{b}$, and then solve for a and b
- Denote the present discounted cash-flow value (as of beginning of the period) of an asset owner with private valuation j by V_j^0 and that of a non-owner by V_j^{no}
- Since non-owner buys if $V_h^o - a \geq V_h^{no}$ and owner sells if $V_l^{no} + b \geq V_l^o$, we get

$$\bar{a} = V_h^o - V_h^{no}, \tag{1}$$

$$\bar{b} = V_l^o - V_l^{no} \tag{2}$$

- So $a = V_h^o - V_h^{no}$ and $b = z(V_l^o - V_l^{no}) + (1 - z)\mu$.
- Finally, we must calculate the value functions

Solving the DGP model (3)

- We will just look at two functions, the rest are similar

$$V_h^o = \frac{1}{1+r} + \frac{(1-\psi)V_h^o}{1+r} + \frac{\psi(1-\phi)V_l^o}{1+r} + \frac{\psi\phi(V_l^{no}+b)}{1+r}$$

$$V_h^{no} = \frac{\psi V_l^{no}}{1+r} + \frac{(1-\psi)(1-p^B)V_h^{no}}{1+r} + \frac{(1-\psi)p^B(V_h^o-\bar{a})}{1+r}$$

- Calculate V_l^o and V_l^{no}. Plug back into (1)-(2) to solve for $\bar{a}$ and $\bar{b}$

DGP: Results (1)

- Ask price is then

$$a = \frac{1}{r} - \frac{2\psi}{r(1+z)}\left(1 - \phi\frac{1-z}{2}\right)S,$$

where S is the spread

$$S = a - b = \frac{(1+z)c}{2(r+2\psi) + (1-2\psi)\phi(1-z)}$$

- If $\psi > 0$ the ask price is less than the $1/r$ which would arise if it were always (efficiently) held by high-value traders
- This is due to liquidity costs: buyers anticipate that they will sell in the future and incur the spread cost

DGP: Results (2)

- The midquote is

$$\mu = \frac{1}{r}\left(1 - \frac{c}{2}\right) - \frac{c}{2r} \cdot \frac{\phi(1-z)}{2r + 4\psi - (2\psi - 1)\phi(1-z)}$$

- Increasing in ϕ: larger ϕ = smaller prob of not finding a trade ϕ = smaller illiquidity risk = more valuable asset