Financial Markets Microstructure
Lecture 2

Prices and Liquidity
Chapter 2 of FPR

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What did we do last week?

1. Introduced financial markets broadly speaking and motivate why we wanted to talk about it
2. Characterized market formats: order-driven markets (auctions) and dealer markets
3. Introduced some of the key concepts and language: dealer sets bid/ask price, traders submit market/limit orders

This lecture:

Fundamental Value

- We'd like to believe there is some "objective" / "fundamental" value of a stock – at least to some representative agent.
- The question of whether to buy of sell then often amounts to:
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  "Is the current asset price above or below its fundamental value?"
- To answer, need to understand what fundamental value is.
- **Short answer**: expected discounted cash flow. Affected by many factors:
  1. R+D
  2. Governance
  3. Marketing
  4. Competition
  5. ...

  Also affected by the representative agent's preferences (discounting, risk-aversion, income profile)

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- Dual question: price discovery
  "How much information about the fundamental value can be extracted from market prices?"

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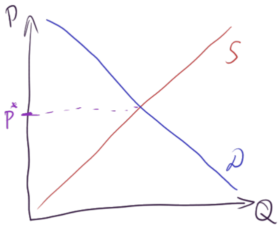
- Long answer: study asset pricing.
  - Field of finance devoted to calculating asset price relative to other assets, assuming perfect markets.
- Question of **market microstructure**: take the 'fundamental value' as given and analyze how it translates into prices in realistic markets.
  - In GameStop case, divergence was due to bubble(?)
  - Another broad reason for divergence: limited liquidity
- Dual question: price discovery
  "How much information about the fundamental value can be extracted from market prices?"

This lecture:

What is liquidity?

- In perfect markets:
  - There is one price = valuation cutoff
  - Agents who value asset above the cutoff end up with it (keep or buy)
  - Agents who value asset less end up without it (sell or do not buy)
  - This is the efficient allocation that we want



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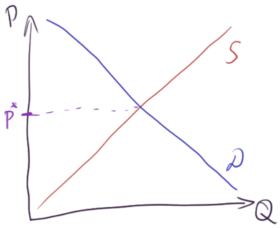
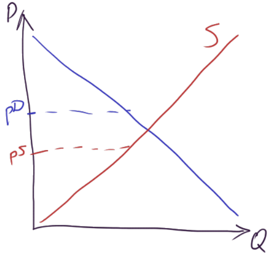
  - This is the efficient allocation that we want
- In (financial and many other) real markets:
  - Bid/ask prices different from that ideal cutoff/fundamental value
  - (due to limited liquidity)
  - Allocation inefficient





Still, what exactly is liquidity?

- Market liquidity = "market's ability to facilitate an asset being sold quickly (for cash) without having to reduce its price very much (or even at all)"
  - Not everyone who wants to trade in a given asset is present in the market at the same time
  - Liquidity depends not just on exogenous parameters, but also traders' endogenous behavior

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- Do not confuse with (related notions of):

1. Funding liquidity = "economic agent's ability to obtain cash/credit at acceptable terms, to meet obligations without incurring large losses"
   - Banks are 'liquidity constrained' when they do not have enough cash on hands to meet demand for withdrawals (despite having enough assets)
   - You are liquidity constrained when your wage arrives in two days but you need to pay your rent today.
2. Monetary liquidity = "asset's ability to be exchanged for goods"
   - Assets in the order of decreasing liquidity: cash, checking deposits, long-term deposits, housing, ...

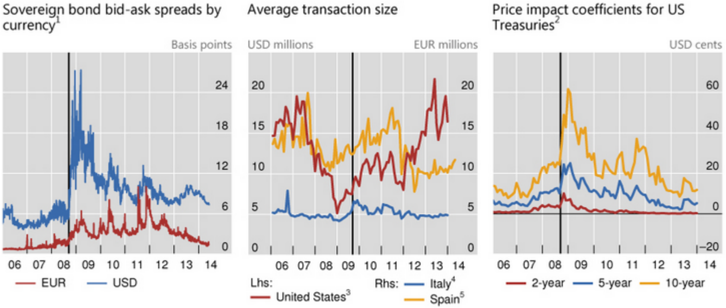
Why do we care about liquidity?

- Traders: liquidity provides a measure of trading costs
- Regulators:
  1. Efficiency is tricky to measure in financial markets: liquidity provides a proxy
  2. Illiquid markets also seem to be more prone to medium-run price deviations from fundamentals
  3. Illiquidity *may* be a sign of structural problems in the market
- Relation to depth: depth measures how much must be traded to move price by certain amount
  - $\approx$ sensitivity of liquidity to trade size

Liquidity dry ups

Liquidity can change over time, can dry up in times of instability

Post-crisis recovery in sovereign bond market liquidity — Graph 1



The black vertical lines correspond to 15 September 2008 (the date of the Lehman Brothers bankruptcy).

[1] Based on Markit iBoxx indices; includes domestic and foreign sovereign bonds denominated in US dollars and euros, respectively. [2] Estimated price change per $1 billion net order flow; monthly averages. [3] Average transaction size for 10-year US Treasury note. [4] Average transaction size on MTS Cash, an inter-dealer market and the most important wholesale secondary market for Italian government bonds. [5] Average transaction size for Spanish public debt.

Sources: CGFS Study Group member contributions based on national data; Markit iBoxx; BIS calculations.

This lecture:

Liquidity measures

Liquidity is not a very well-defined concept, so it's not immediate how to measure it either. We will consider several measures:

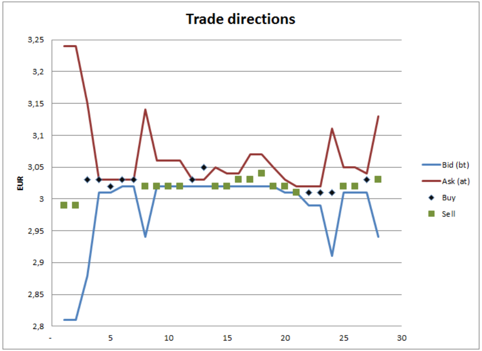
1. **Spread measures**: quoted spread, effective spread, realized spread
2. **Volume-weighted average price**: simply using average prices
3. **Price impact**: How much does the price move after a trade
4. **Non-trading measures**: trading volumes

When trade direction is not available: estimate via Lee-Ready algorithm.

When quote data are not available: estimate them using Roll's measure

Example

- We will play around with a dataset on KrispyKreme stock



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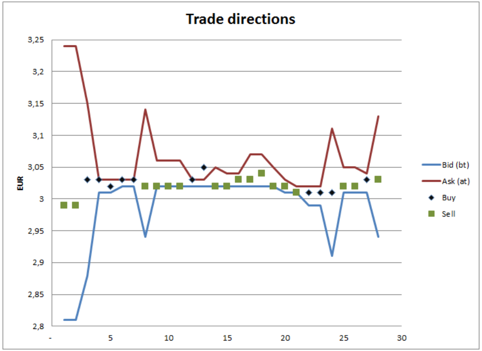

- Notice that price is sometimes inside spread: price improvements (either hidden limit orders or price improvement given by dealer)
- Also a price outside spread (recall bid/ask only valid for x units)

Quoted spread

- Let a_t (b_t) be the best ask (bid) price at time t
- Quoted spread:

$$S_t = a_t - b_t$$

- Contemporaneous: spread facing trader at time t

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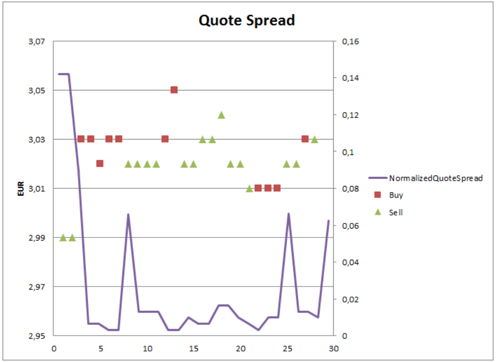
- We can generalize it to consider average spread for trade size q:

$$S_t(q) = \overline{a}_t(q) - \overline{b}_t(q)$$

where $\overline{a}_t(q)$ and $\overline{b}_t(q)$ are average execution prices

Quoted spread

- Applying the definition to the data, we get:



- Notice that the quoted spread does not capture price improvements (for instance in the first three observations)

Effective spread

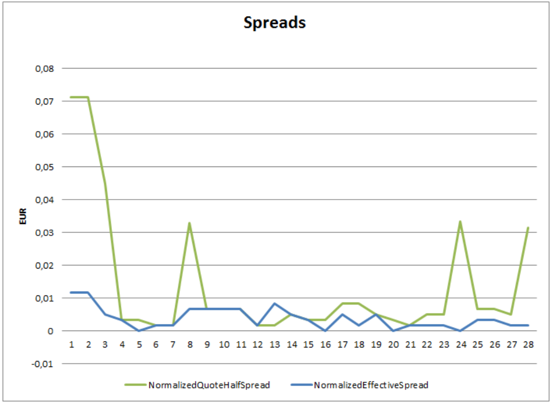
- Suppose one market order is executed per period, and
  - d_t: trade direction (1: buyer-initiated, -1: seller initiated)
  - p_t: price
- Effective (half-)spread:

$$S_t^e = d_t(p_t - m_t),$$
$$s_t^e = \frac{S_t^e}{m_t}$$

- Backward looking: spread faced by previous trader
- Compare actual price with midquote the instant before: measures price impact and captures 'price improvements'

Effective spread

- Apply to data and compare to quoted spread



- Effective spread is often lower (since it captures price improvements); can be larger if there was a big market order

Realized spread

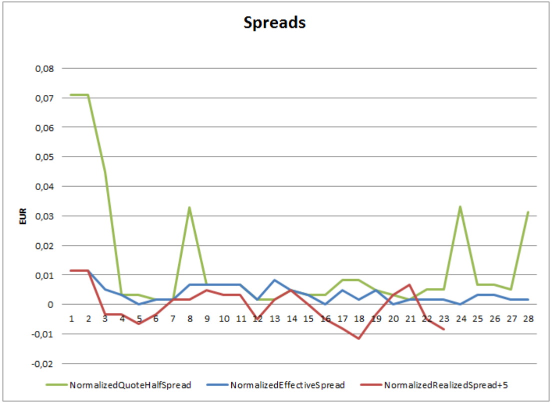
- Realized spread:

$$\begin{aligned} S_t^r &= d_t(p_t - m_{t+\Delta}) \\ &= d_t(p_t - m_t) - d_t(m_{t+\Delta} - m_t) \end{aligned}$$

- Spread realized by somebody who holds the asset for Δ periods
- Idea: measure the spread after prices have adjusted to new information
- As a forward-looking measure:
  - $\mathbb{E}_t S_t^r = d_t(p_t - m_t) = S_t^e$ if $\mathbb{E}_t m_{t+\Delta} = m_t$
- As a backward-looking measure:
  - Typically smaller than effective spread: why?

Realized spread

- Calculate for $\Delta = 5$ and compare to other measures



- Realized spread is indeed often lower than effective spread

Comparing the spreads

- The quoted spread and the effective spread may be more useful to traders:
  - Quoted spread: what is the quoted trading cost now
  - Effective spread: what was the trading cost faced by the last trader

  These are (imperfect) measures of the cost of executing a market order now
- The realized spread is more relevant to a market maker (liquidity provider):
  - It measures the cost of taking a position (long or short) for an amount of time

This lecture:

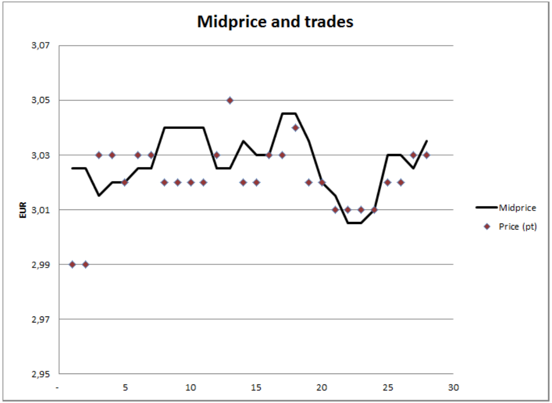
Estimating direction of trade

- We often only observe quotes and realized prices: not the direction of trade
- Thus, we need to develop methods to classify trades
- Complication: trading may be 'within the quotes': harder to guess direction
- Lee-Ready algorithm: (Lee and Ready [1991])

$$d_t = \begin{cases} 1 \text{ if } |p_t - a_t| < |p_t - b_t| \\ \quad \text{or } p_t = m_t \text{ and } p_t > p_{t-1} \\ -1 \text{ if } |p_t - a_t| > |p_t - b_t| \\ \quad \text{or } p_t = m_t \text{ and } p_t < p_{t-1} \end{cases}$$

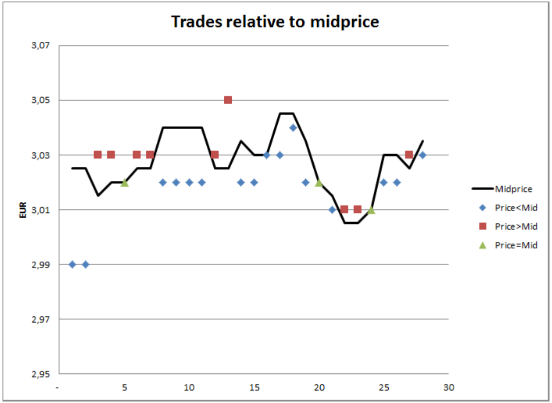
Estimating Lee-Ready

First, we calculate midprices and compare to trade prices



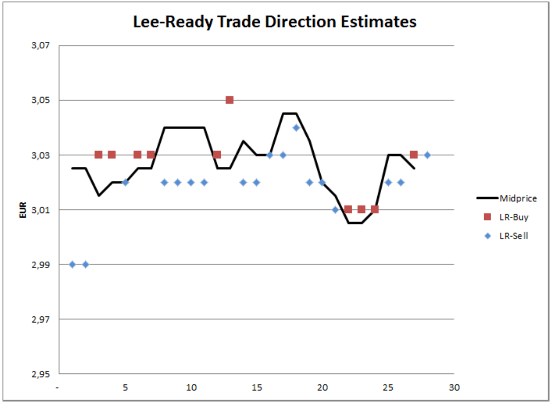
Estimating Lee-Ready (2)

Then we compare trade prices to midprices



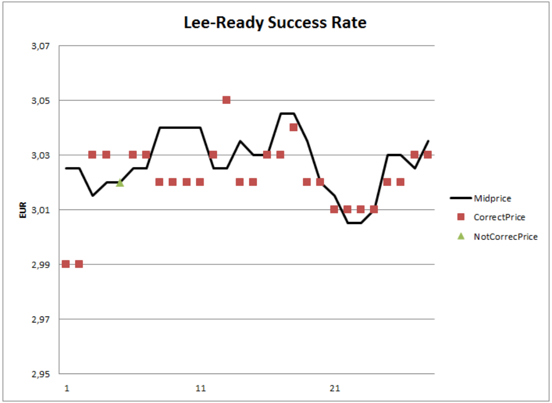
Estimating Lee-Ready (3)

Finally, we classify the trades that were just on the midprice



Estimating Lee-Ready (4)

Checking with the actual trade directions, we see that we only made one mistake



Lee-Ready precision

- Odders-White [2000]: large-scale (144 NYSE stocks over 3 months; $>$ 400k transactions) test of Lee-Ready algorithm with NYSE data
- 85% correct
- Most mistakes with:

Lee-Ready precision

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- 85% correct
- Most mistakes with:
  - trades at the midpoint
  - small transactions
  - transactions in large-cap / frequently traded stocks

This lecture:

1. Fundamental value and Prices
2. Liquidity
3. Measures of Liquidity
4. Estimating Direction of Trade
5. **Estimating Quotes**
6. Other Measures of Liquidity

Quote data

- We often lack information on quotes to compute the spread
  - Can we estimate the spread knowing only trade prices?
- Roll [1984]: use transaction prices to estimate it
  1. Construct a simple model of trading and calculate spread
  2. Estimate it
  3. Check robustness to simplifying assumptions

Roll's model

Suppose the following:

1. All trades have the same size. $d = 1$: buy, $d = -1$: sell
2. Arriving orders are i.i.d. with $\mathbb{P}(d_t = 1) = \frac{1}{2}$
3. Midquote is random walk: $m_t = m_{t-1} + \epsilon_t$, where ϵ_t are i.i.d. shocks
4. Market orders are not informative: $\mathbb{E}(d_t \epsilon_t) = \mathbb{E}(d_t \epsilon_{t+1}) = 0$
5. Spread $S = a_t - b_t$ is constant.

Then

$$p_t = m_t + \frac{d_t S}{2}.$$

We know p_t but not m_t. How do we estimate S?

Roll's model

- Roll's observed that although ϵ_t and d_t are i.i.d., $\Delta d_t = d_t - d_{t-1}$ is mean-reverting:

$$Cov(\Delta d_t, \Delta d_{t-1}) = -1$$

- Intuitively: if $\Delta d_t > 0$, this means that we go from a sale to a buy - then the next change must be opposite

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&= \frac{S^2}{4}\mathbb{E}[-d_{t-1}^2] = \frac{-S^2}{4}
\end{aligned}
$$

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The estimator

- We can then work out that

$$Cov(\Delta p_t, \Delta p_{t-1}) = -\frac{S^2}{4},$$

  giving us the estimator

$$S_t^R = 2\sqrt{-Cov(\Delta p_t, \Delta p_{t-1})}.$$

- Recall the assumptions of the model. We (the book) can work out extensions to treat some of them
- In our example: $S_t^R = 0.01$

This lecture:

Price impact

- How much do trades affect prices? Price impact λ; $1/\lambda$ captures market *depth*

$$\Delta m_t = \lambda q_t + \epsilon_t.$$

Here q_t is the order imbalance in period t. In our example: $\lambda = 0.15$ (q_t in 100,000EUR)

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- Hasbrouck measure (γ): sensitivity of returns to trading volume (Hasbrouck [2007])

$$|\Delta m_t| = \gamma Vol_t + \epsilon_t.$$

Meaningful for single trades, but if t aggregates many trades, γ is hard to interpret.

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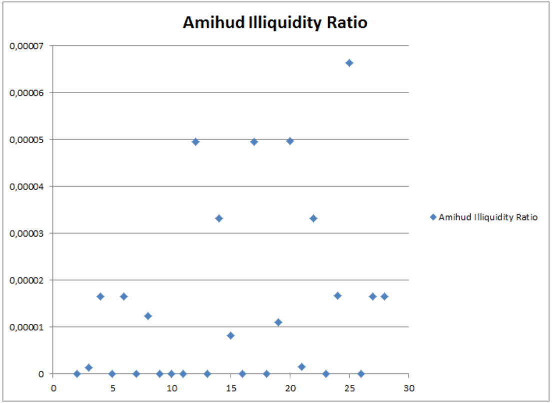
- Amihud measure (I_t): take ratio btw return Δm_t and volume to get *illiquidity ratio*: (Amihud [2002])

$$I_t = \frac{|\Delta m_t|}{Vol_t}.$$

Amihud's Illiquidity Ratio



Somewhat volatile on high-frequency data, usually taken as an average over longer intervals (month) – but then it is also hard to interpret.

On Hasbrouck and Amihud measures

- Neither Hasbrouck, nor Amihud measures make immediate sense when applied to aggregate data – yet this is the most common application.
- Afaik, at least the Hasbrouck measure was born to deal with pre-1983 historical stock data, which only contained aggregated daily prices and volumes, and no intraday data.
- Hasbrouck [2007] shows that γ is mildly correlated with λ under some distributional assumptions
- Bottom line: do not use γ or I if you have data that lets you estimate λ directly.

Volume based measure

- Volume-Weighted Average Price (VWAP):

$$VWAP = \sum w_i p_i,$$

  where $w_t = |q_i| / \sum_i |q_i|$ is the order weight, q_i is the size of order i

- This equals the amount of dollars traded over the number of shares traded: average price
- Trader can compare the price he got with VWAP to evaluate how good was his deal relative to market.
- This measure may depend excessively on few orders (if they are large) and therefore be subject to manipulation
- For our example, $VWAP = 3.02$

Implementation shortfall

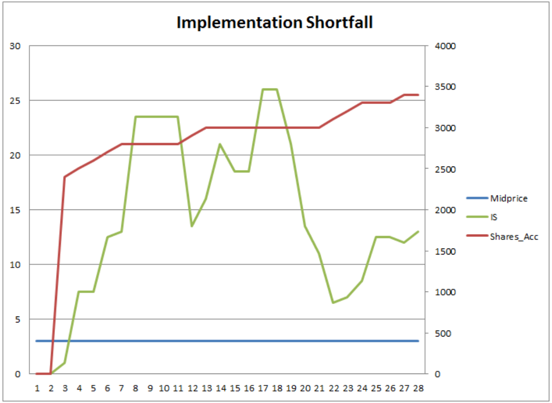
- Aim at time 0: to (net) purchase q shares
  - By time t, fraction κ_t has been executed, at an average execution price $\bar{p}_t$
  - The realized trading gain is $\kappa_t q(m_t - \bar{p}_t)$
  - An ideal gain from immediate execution without price impact would have been $q(m_t - m_0)$
  - The difference is the implementation shortfall:

$$\begin{aligned} IS_t &= q(m_t - m_0) - \kappa_t q(m_t - \bar{p}_t) \\ &= \kappa_t q(\bar{p}_t - m_0) + (1 - \kappa_t) q(m_t - m_0). \end{aligned}$$

  - Interpretation: Execution cost plus opportunity cost

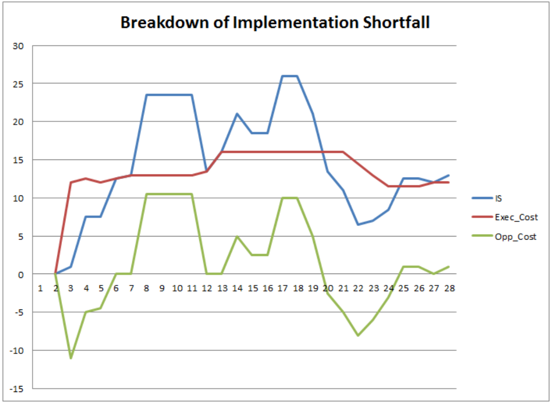
Implementation shortfall

- Suppose in the KrispyKreme example you want to buy 3,500 shares
- And suppose all the buy orders in the data (3,400 shares) came from you



Implementation shortfall

Breaking down the shortfall into opportunity cost and execution cost



Other measures

- Measures such as trading volume and trade frequency are also used
- Time-to-trade for limit orders is another measure, but difficult to use (depends on order size, depends on past traders' intent – some post LOs to provide liq-ty, trading is not the final goal)
- Some measures may contradict each other, e.g.:
  - trading volume and spreads are both positively correlated with information releases (why?)
  - price volatility is low in very liquid – but also very illiquid markets
- Frequency of trading or related measures may be more relevant in 'thin' markets, for instance in emerging economies

Conclusion

- We have looked at different manners in which to estimate liquidity
- No method is perfect: depends on trade size, time horizon, trade motivation
- Data shows that liquidity varies both continuously throughout a trading day, and more abruptly around big events
- Next time we will start analyzing *what* drives the spread

Exercises for next week

- Recreate the graphs and figures and numbers I presented today using the KrispyKreme dataset. Better: calculate the (average) values of all measures for the whole dataset.
- Solve exercise 8 regarding implementation shortfall, on page 75 in the textbook. Discuss the meaning of m_t in this analysis.

References I

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