Financial Markets Microstructure

Lecture 4

Informative Order Flow

Chapter 3.1-3.3 of FPR

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What did we do last week?

1. Discuss liquidity
2. Talk about different measures of:
   - spreads (quoted, effective, realized)
   - depth and price impact
   - execution costs
3. Estimators for missing data
   - Lee-Ready algorithm for estimating trade direction
   - Roll's spread estimator requiring only price data

Overview

Before we start, let's look at an overview of what we'll do in the rest of the course

Part 1: Setting up the models

- Dealer models 1; Glosten-Milgrom, fixed trade size; Bayes' Rule; adverse selection
- Dealer models 2; Kyle, variable trade size; market depth; estimating liquidity determinants, inventory risk
- Limit order book; Glosten: static, random market order demand; market design; Parlour: dynamic, endogenous market order demand

Overview

Part 2: Applying the models

- Fragmentation: costs and benefits
- Transparency: search costs and order flow transparency
- Value of liquidity
- Liquidity and corporate policy

Overview

Part 3: Topics and specialized models

- High-frequency trading (Biais et al., Budish et al.)
- Public information; optimal disclosure policies; public announcements and trade volumes (Kondor)
- Bubbles; herding; common knowledge (Smith and Sørensen, Abreu and Brunnermeier)

Today

1. Beginning of a long discussion about determinants of the spread
2. Go in-depth with the relation between information and prices
3. What is informational efficiency?
4. Discuss Glosten and Milgrom's model of information-based trading

This lecture:

1 Information and Prices

2 Glosten-Milgrom model

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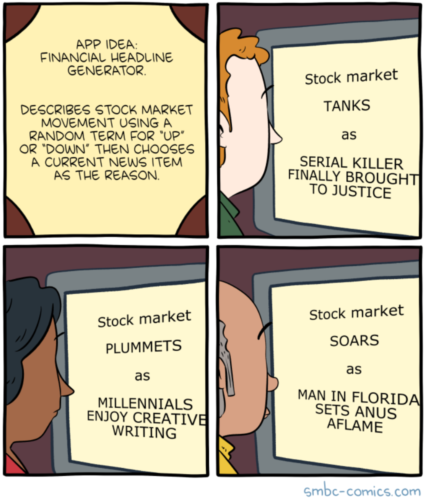
- So what drives the spread? (apart from market thinness)
  - To answer that, first talk about what drives the prices.

Price determinants

- Stock prices move around all the time
- Media reports are full of ex post rationalizations...
- ...but these can be pretty arbitrary →
- What actually determines stock prices and their movement?



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- traders have different valuations (**why?**) $\rightarrow$ prices clear demand and supply

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$\Rightarrow$ Reasons for trading:

- Traders have different value from the same cash flows
  - **Risk**: Getting the right risk profile
  - (Funding) **Liquidity**: Trader needs liquid funds or has excess funds to invest
- Traders have **private information** about the fundamentals

*IRL, people are very noisy in decision-making and disagree a lot with each other and even with past selves given the same info [Kahneman, Sibony, and Sunstein, 2021]

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- Reminder: we draw a wedge between legal and academic ($\leftarrow$ our) definitions of **private information**
  - Legal: insider info is that which is only available to a restricted number of people. Trading based on insider info is illegal in most jurisdictions.
  - Academic: some agents may be better at analyzing publicly available info, thus have better info about fundamentals.

Information and prices

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Fama's **efficient markets hypothesis**: prices must be efficient.
I.e., they must reflect all available information. Example: next slide

How Dr. Alchian learned to build the bomb

We knew they were developing this H-bomb, but we wanted to know, what's in it? What's the fissile material? Well there's thorium, thallium, beryllium, and something else, and we asked Herman Kahn and he said, 'Can't tell you'... I said, 'I'll find out', so I went down to the RAND library and had them get for me the US Government's Dept. of Commerce Yearbook which has items on every industry by product, so I went through and looked up thorium, who makes it, looked up beryllium, who makes it, looked them all up, took me about 10 minutes to do it, and got them. There were about five companies, five of these things, and then I called Dean Witter... they had the names of the companies also making these things, 'Look up for me the price of these companies...' and here were these four or five stocks going like this, and then about, I think it was September, this was now around October, one of them started to go like that, from $2 to around $10, the rest were going like this, so I thought 'Well, that's interesting'... I wrote it up and distributed it around the social science group the next day. I got a phone call from the head of RAND calling me in, nice guy, knew him well, he said 'Armen, we've got to suppress this'... I said 'Yes, sir', and I took it and put it away, and that was the first event study.

–from a 2000 interview with A.Alchian, transcribed by Newhard [2014]

Types of informational efficiency

There are different kinds of price efficiency:

- **Weak form**: Prices reflect historic (price) information
- **Semi-strong form**: Prices reflect all public information
- **Strong form**: Prices reflect all public and private information

Strong is the kind of efficiency we had in mind in the previous lectures. It arises in classical models in Economics (see General Equilibrium theory)

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  - Suppose every agent has some private signal about the fundamental, but also knows that the price incorporates all private info of all agents. Then it's optimal to ignore own signal, since it adds nothing to the price signal. But if everyone ignores all signals, price cannot be informative.

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So we will not take EMH for granted, but rather see whether it arises in models endogenously.

Asset value

- Let's try to figure out how price efficiency looks like in math.
- Information: Ω_t captures the market's (public) knowledge at time t: $\Omega_{t+1} = (\Omega_t, I_{t+1})$ (I_{t+1} is the new info + assume perfect memory).
- Market valuation ($\neq$ price) of an asset = expectation of underlying fundamental value v given information Ω_t:

$$\mu_t = \mathbb{E}\left[v|\Omega_t\right].$$

Think of v as the sum of discounted cash flows: $v = \sum_{s=t}^{\infty} \delta^{s-t} c_s$ (uncertain as of t).

Informational efficiency (1)

- Informational efficiency then corresponds to: $p_t = \mu_t = \mathbb{E}[v|\Omega_t]$
  (Ω_t is "market's knowledge", so efficiency understood as semi-strong)
- Valuation only changes if new information arrives: 'innovation in value' is a random variable: $\epsilon_{t+1} = \mu_{t+1} - \mu_t$. Then

$$
\begin{aligned}
\mathbb{E}[\epsilon_{t+1}|\Omega_t] &= \mathbb{E}[\mu_{t+1} - \mu_t|\Omega_t] \\
&= \mathbb{E}[\mu_{t+1}|\Omega_t] - \mathbb{E}[\mu_t|\Omega_t] \\
&= \mathbb{E}[\mathbb{E}[v|\Omega_{t+1}]|\Omega_t] - \mu_t \\
&= \mathbb{E}[v|\Omega_t] - \mu_t \\
&= \mu_t - \mu_t \\
&= 0.
\end{aligned}
$$

Informational efficiency (2)

- Also, $\mathbb{E}[\epsilon_s \epsilon_t] = 0$, $\forall s \neq t$.
- Price innovation is equal to the valuation innovation:

$$p_{t+1} - p_t = \mu_{t+1} - \mu_t = \epsilon_{t+1}$$

- Thus

$$\mathbb{E}[p_{t+1} | \Omega_t] = p_t$$

- When we have informational efficiency, the price is a martingale

This lecture:

Glosten and Milgrom [1985]

> *All models are wrong; some models are useful.*
>
> – George Box

"This not completely accurate model of mammoth. Am only meant build up intuition."

GM85: Overview

- The simplest model explaining spread through private information.
- Spread is driven by adverse selection

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- Dynamic model, periods $t = 1, 2, \ldots$;
  (though we will be analyzing the stage game for a given period – essentially static)
- Two players in every period:
  - trader and dealer
  - dealer long-lived; trader new every period
  - trader can be informed or not
- One asset with fundamental value v (unknown), common belief $v \sim F(v)$

GM85: Model (1)

Trader: is either a speculator or a noise trader, can submit a market order $d_t \in \{1, -1\}$ to buy or sell one unit of the asset with fundamental value v (or do nothing, $d_t = 0$)

- Speculator (probability π): has private information about v.
  - We will usually assume speculator simply knows v (not much changes if he only has a noisy private signal about it).
  - Risk neutral, chooses his market order d_t to maximize expected profit $d_t \cdot (v - p_t)$:
- Noise trader (probability $1 - \pi$): no pvt info about v; trades for other reasons (hedging, liquidity).
  - We assume he follows some fixed strategy: buys with probability β_B; sells w.p. β_S; abstains w.p. $1 - \beta_B - \beta_S$
  - Important: this assumption is for simplicity only; this strategy can be perfectly rational! We just don't model what generates it.
    E.g., could say noise traders choose d_t to maximize profit $\mathbb{E}[d_t(v + y_t - p_t)]$, where y is t-trader's idiosyncratic valuation (due to risk, time, liquidity preferences...)

GM85: Model (2)

Dealer (market maker)

- Risk neutral
- Willing to trade exactly one unit (buy/sell/no trade) each period
- Sets bid and ask prices (for a single unit)
- Quote price before seing trade (limit order)
- Does not know whether trader is speculator or noise trader (but knows π)
- Expected profit from trade is $\mathbb{E}[-d_t(v - p_t)]$
- Competitive: prices=expected asset value conditional on information
- Trading is sequential: market orders served one by one

Aside on Dealers

"For each security in which a member is registered as a Market Maker, the member shall be willing to buy and sell such security for its own account on a continuous basis during regular market hours and shall enter and maintain a two-sided trading interest ("Two-Sided Obligation") that is identified to the Exchange as the interest meeting the obligation and is displayed in the Exchange's quotation montage at all times."

– Nasdaq Rule 4613

"Minimum requirements: at least 85% of the time, at most 4% bid-offer spread, order size at least worth 4,000 euros"

– Nasdaq OMX Helsinki

GM85: Model (3)

- **Equilibrium**:
  - An equilibrium consists of bid and ask prices and speculator's strategy
  - They must be such that: (i) prices are competitive (zero profit for MM), (ii) speculator best-responds to prices (maximizes expected gain).

Analysis. A: Market making

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- Perfect competition among dealers implies zero expected profit from either trade type $\Rightarrow$ ask price and bid price are

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- Notice that both sides of the equality depend on prices

Analysis. B: Informed trading

- Speculator knows v. Given prices a_t and b_t, the expected profits Π are:

$$\Pi(v, a_t, b_t, d_t) = \begin{cases} v - a_t & \text{if } d_t = 1; \qquad (\mathit{Buy}) \\ 0 & \text{if } d_t = 0; \qquad (\mathit{Abstain}) \\ b_t - v & \text{if } d_t = -1. \qquad (\mathit{Sell}) \end{cases}$$

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- Speculator's best response to (a_t, b_t) is: (assume $a_t \geq b_t$)
  - Buy when $v > a_t$, i.e. when v is large enough
  - Sell when $v < b_t$, i.e. when v is small enough
  - Abstain if $a_t > v > b_t$

Analysis. C: Equilibrium definition

Dealer must make zero profit (competition), traders must trade optimally. This gives us two equilibrium conditions.

- Let σ_t denote the speculator's strategy, where $\sigma_t(d_t|v)$ is the probability that the speculator places order d_t if value is v
- An equilibrium consists of prices (a_t, b_t) and strategy σ_t such that:
  1. the ask and bid prices solve

$$a_t = \mathbb{E}[v|\Omega_{t-1}, Buy];$$
$$b_t = \mathbb{E}[v|\Omega_{t-1}, Sell],$$

     given σ_t
  2. for each v, σ_t solves

$$\max_{\sigma_t} \{\sigma_t(1|v)[v - a_t] + \sigma_t(-1|v)[b_t - v]\},$$

     given (a_t, b_t).

To be continued...

- We will finish the derivation of the general case next time.
- But the knowledge so far should be sufficient to solve the example:

GM Example

- **Single period**: Suppose only one period (drop t subscript, drop Ω)
- **Binary outcome**: $v \in \{0, 1\}$, equally likely ex ante: $\mathbb{P}(v = 1) = 0.5$.
- Suppose $0 < b < a < 1$ and noise trader's order obeys $\beta_B = \beta_S = 0.5$.

Questions:

1. What is the speculator's trading strategy?
2. Can you derive dealer's prices a and b, as a function of π?
   - If not, refresh your knowledge of conditional expectations and try again.
   - If you already read the solution in the book, try to replicate it without looking back at the book.
3. Are the resulting prices efficient? (Check all three forms)

References I

L. R. Glosten and P. R. Milgrom. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14(1):71–100, March 1985. ISSN 0304-405X.

S. J. Grossman and J. E. Stiglitz. On the impossibility of informationally efficient markets. *The American economic review*, 70(3):393–408, 1980. Publisher: JSTOR.

D. Kahneman, O. Sibony, and C. R. Sunstein. *Noise: a flaw in human judgment*. Hachette UK, 2021.

P. Milgrom and N. Stokey. Information, trade and common knowledge. *Journal of Economic Theory*, 26(1):17–27, February 1982. ISSN 0022-0531. doi: 10.1016/0022-0531(82)90046-1.

J. M. Newhard. The stock market speaks: How Dr. Alchian learned to build the bomb. *Journal of Corporate Finance*, 27:116–132, August 2014. ISSN 0929-1199.