Financial Markets Microstructure: Problem Set 2

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Problem 1

Trading at exchanges sometimes breaks down due to technical problems. E.g., The Economist reported on August 31, 2013: "On August 26th trading on Eurex, the main German derivatives exchange, opened as usual; 20 minutes later it shut down for about an hour. Four days earlier the shares of every company listed on NASDAQ, an American stock exchange, ceased trading for three hours".

What are the implications of such breakdowns for liquidity risk? How do they affect asset prices? How does competition among exchanges affect breakdown frequency?

Problem 2 [Ch.7, ex.4]

This problem deals with competition between limit order markets with uniformly distributed market orders. Consider the model of section 7.4.2 ("Glosten model with fragmented market" from Lectures) and assume that the size of the market order ($\tilde{X}$) has a uniform distribution $[0, \bar{X}]$. That is, $F(x) = x/\bar{X}$. We denote by $Y^*_{jk}(\gamma)$ the cumulative depth posted at the ask price $A_k = \mu + k\Delta$ in market $j \in \{I, E\}$ when the fraction of investors submitting market orders in both markets I and E is γ, and by c_j be the submission cost in market j.

(a). Assume that $2c_I \leq \Delta$ and that $\gamma = 0$. Show that the equilibrium cumulative depth at price A_1 is[1]

$$Y^*_{I1}(0) = \bar{X}\left(1 - \frac{2c_I}{\Delta}\right).$$

Hint: Use (7.13).

(b). Now suppose that γ is high enough and that the other parameters are such that $Y^*_{I1}(\gamma) > 0$, $Y^*_{E1}(\gamma) > 0$, but $Y^*_{I1}(\gamma) + Y^*_{E1}(\gamma) < \bar{X}$. Compute $Y^*_{I1}(\gamma)$ and $Y^*_{E1}(\gamma)$ as a function of γ.

[1] The A_1 was misprinted as A_k in the problem text in the book.

Deduce further from the result that the conditions $Y^*_{I1}(\gamma) > 0$ and $Y^*_{E1}(\gamma) > 0$ are satisfied if $\frac{4c_I}{\Delta(2-\gamma)+2c_E} < 1$ and $\frac{4c_E}{\Delta+2c_I} < \gamma$. Moreover, deduce that the condition $Y^*_{I1}(\gamma) + Y^*_{E1}(\gamma) < \bar{X}$ is satisfied if $4(\gamma c_I + (2-\gamma)c_E) > (2-\gamma)\gamma\Delta$.

Hint: You need the equations (7.11), (7.12), (7.14) and (7.15) to get the system of equations that pins down Y_I and Y_E. You can then either solve the algebra by muscle (or use some computer algebra system[2] to provide that muscle for you) or try to rewrite the two equilibrium conditions so as to eliminate either Y_I or Y_E.

(c). Deduce from question (b) that the two markets can coexist even if their order submission costs differ and $\gamma = 1$.

Hint: first think about the case where $c_I = c_E = c$. This will give you an interval for Δ in which the markets can coexist. Then argue that there exist some $c_I \neq c_E$ such that this is true as well.

(d). Why does the cumulative depth at price A_1 in one market decrease with the order submission cost in this market but increase with the cost in the competing market?

(e). Consider the case $\gamma = 1$ and suppose that $4(c_I + c_E) < \Delta$ and $4c_I < \Delta$. Compute $Y^*_{I1}(1)$ and $Y^*_{E1}(1)$.

Hint: Notice that now we are violating one of the conditions given in (b). What effect does this have on $F(Y_I + Y_E)$? Take account of this when writing up (7.14) and (7.15).

(f). Under the assumptions in question (e), what is the number of shares offered at price $A_k > A_1$? Is the result different when $\gamma = 0$?

Hint: Look at the values of Y^*_{I1} in the two cases.

Problem 3

MiFID II, the recent European financial market regulation, requires that "firms shall disclose to the client information on the payment or benefit concerned, in a manner that is comprehensive, accurate and understandable" (in accordance with the second paragraph of Article 24(9) of MiFID II). Evaluate the possible effects of this regulation.

In particular, suppose that some asset is traded at multiple exchanges. One of the exchanges offers one of the banks a payment for directing order flow originating from bank's clients towards this exchange. This relates either to all order flow, or to order flow from retail investors. How would the bank's obligation to be transparent about this fee towards its clients affect market outcomes?

[2] A popular choice is Wolfram Alpha available at `https://www.wolframalpha.com`. I personally prefer open-source alternatives like (wx)Maxima (`https://wxmaxima-developers.github.io/wxmaxima/`) or SageMath (`https://www.sagemath.org/`).